

Genesis:
How the Universe Began
According to
Standard Model Particle Physics

by

Frank J. Tipler¹

Department of Mathematics and Department of Physics
Tulane University
New Orleans, Louisiana 70118 USA

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¹ e-mail address: TIPLER@TULANE.EDU

Abstract

I show that the mutual consistency of the Bekenstein Bound, the Standard Model (SM) of particle physics, and general relativity implies that the universe began in a unique state, an initial Friedmann S^3 singularity at which the temperature, entropy, Higgs field, baryon number, and lepton number were zero, but with a non-zero SU(2) (gravitational) sphaleron field. I solve the coupled EYM equations for this unique state, show how the horizon problem is solved, and how SM baryogenesis naturally results from the triangle anomaly. Since the SU(2) winding number state is thus non-zero, the universe is not in the QCD ground state, and this plausibly yields a (small) positive cosmological constant. Since the initial state is unique, it is necessarily homogeneous and isotropic, as required by the Bekenstein Bound. Wheeler-DeWitt quantization implies an S^3 cosmology must be very close to flat if the universe is to be classical today. I show that the spectrum of any classical gauge field (or interacting massless scalar field) in a FRW universe necessarily obeys the Wien displacement law and the corresponding quantized field the Planck distribution law with the reciprocal of the scale factor R playing the role of temperature, even if the fields have zero temperature. Thus the CBR could even today be a pure SU(2) electroweak field at zero temperature coupled to the Higgs field, in spite of early universe inverse double Compton and thermal bremsstrahlung. I conjecture that such a pair of fields with this coupling can yield a weakly interacting component with mass density decreasing as R^{-3} and an EM interacting component with mass density decreasing as R^{-4} , the former being the dark matter and the latter the CBR. Except that such a CBR would not couple to right-handed electrons, and this property can be detected with a Penning trap or even using the late 1960's CBR detector with appropriate filters. I argue that right handed ultrahigh energy cosmic ray protons would not produce pions by interacting with such a CBR, and thus the existence of such protons may constitute an observation of this CBR property. I show that such a CBR has no effect on early universe nucleosynthesis, and no effect on the location of the acoustic peaks.

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1. Introduction

It is generally agreed that the non-zero baryon number of the universe requires explanation. Most baryogenesis scenarios envisage some baryon number violating process occurring at high temperature, typically at the GUT scale or at the Planck temperature of 10^{19} GeV. But a high temperature implies a high entropy density σ , since $\sigma \propto T^3$, and thus these scenarios leave open the question of where the entropy came from. For the most natural initial value of the universal entropy is zero, just as the most natural initial value of the baryon number is zero.

I shall show that the physical laws impose their own unique boundary condition on the universe, requiring the universe to begin in a unique state of zero entropy, zero temperature, zero baryon number and zero lepton number. Such a universe must be topologically S^3 , initially perfectly isotropic and homogeneous, with zero Higgs field, but with a non-zero SU(2) sphaleron field. I shall give the exact solution to the Einstein-Yang-Mills equations for this unique initial state, and show how it evolves. I shall propose two experiments to test my model for the early universe.

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2. Apparent Inconsistencies in the Physical Laws in the Early Universe

a. Bekenstein Bound Inconsistent with Second Law of Thermodynamics

The fundamental limitation on the number of possible quantum states in a bounded region — or, alternatively, on the number of bits that can be coded in a bounded region — is given by the Bekenstein Bound [1,2]. The Bekenstein Bound is a consequence of the basic postulates of quantum field theory. A derivation will not be given here, but in essentials the Bekenstein Bound is just another manifestation of the Heisenberg Uncertainty Principle.

If, as is standard, the information I is related to the number of possible states N by the equation $I = \log_2 N$, then the Bekenstein Bound on the amount of information coded within a sphere of radius R containing total energy E is

$$I \leq 2\pi ER/(\hbar c \ln 2) \quad (2.1)$$

or, expressing the energy in mass units of kilograms,

$$I \leq 2.57686 \times 10^{43} \left(\frac{M}{1 \text{ kilogram}} \right) \left(\frac{R}{1 \text{ meter}} \right) \text{ bits} \quad (2.2)$$

For example, a typical human being has a mass of less than 100 kilograms, and is less than 2 meters tall. (Thus such a human can be inscribed in a sphere of radius 1 meter.) Hence, we can let M equal 100 kg and R equal 1 meter in formula (2.2) obtaining

$$I_{\text{human}} \leq 2.57686 \times 10^{45} \text{ bits} \quad (2.3)$$

as an upper bound to the number of bits I_{human} that can be coded by any physical entity the size and mass of a human being.

Let me give an elementary *plausibility argument* for the Bekenstein Bound (2.1). This argument will be nonrigorous. (A completely rigorous proof would involve too much quantum field theory to be feasible in this book.) The Uncertainty Principle tells us that

$$\Delta P \Delta R \geq \hbar \quad (2.4)$$

Where ΔP is the ultimate limit in knowledge of the momentum and ΔR is the limit in knowledge of the position. (Alternatively, the inequality (2.4) expresses the minimum size of a phase space division.) Thus, if the total momentum is less than P and the system is known to be inside a region of size R , then the phase space of the system must be divided into no more than $PR/\Delta P \Delta R = 2\pi PR/\hbar$ distinguishable subintervals. This means that the number of distinguishable states n is bounded above by $2\pi PR/\hbar$. Since for any particle, $P \leq E/c$, where E is the total energy of the system including the system's rest mass, with equality holding only if the system is moving at the speed of light, we have

$$I = \log_2 n \leq \frac{n}{\ln 2} \leq 2\pi \left(\frac{E}{c} \right) \left(\frac{R}{\hbar \ln 2} \right) \leq \frac{2\pi ER}{\hbar c \ln 2}$$

which is the Bekenstein Bound (2.1). (Additional complications like particle substructure, and the fact that the system is in three dimensions rather than one are implicitly taken into account by the fact that $\log_2 n$ is very much less than n , for large n . As I said, the above derivation is nonrigorous.)

An upper bound to the information processing rate can be obtained [1] directly from the Bekenstein Bound by noting that the time for a state transition cannot be less than the time it takes for light to cross the sphere of radius R , which is $2R/c$. Thus

$$\dot{I} \leq \frac{I}{2R/c} \leq \frac{\pi E}{\hbar \ln 2} = 3.86262 \times 10^{51} \left(\frac{M}{1 \text{ kilogram}} \right) \text{ bits/sec} \quad (2.5)$$

where the dot denotes the proper time derivative. By inserting 100 kilograms for the value of M in inequality (2.5), we obtain an upper bound for the rate of change of state of a human being, \dot{I}_{human} :

$$\dot{I}_{human} \leq 3.86262 \times 10^{53} \text{ states/sec} \quad (2.6)$$

The significant digits in the RHS of inequalities (2.2), (2.3), (2.5), and (2.6) have to be taken with a grain of salt. The digits correctly express our knowledge of the constants c and \hbar . But the Bekenstein Bound is probably not the least upper bound to either I or \dot{I} ; Schiffer and Bekenstein have recently shown [2] that the Bekenstein Bound probably overestimates both I and \dot{I} by a factor of at least 100.

Strictly speaking, (2.5) only applies to a single communication channel [3], but it probably [4] applies even to multichannel systems if the need to merge the information from various channels is taken into account. However, if the latter is not taken into account, the number of channels is certainly limited by the number of states given by (2.1), and so an extremely conservative upper bound is $\frac{dI}{d\tau} \leq e^{I_{max}^B} \dot{I}_{max}^B$ (J. D. Bekenstein, private communication).

A human being — indeed, any object existing in the current universe — actually codes far less than quantum field theory would permit it to code. For example, a single hydrogen atom, if it were to code as much information as permitted by the Bekenstein Bound, would code about 4×10^6 bits of information, since the hydrogen atom is about one Ångström in radius, and has a mass of about 1.67×10^{-27} kilograms. So a hydrogen atom could code about a megabyte of information, whereas it typically codes far less than a bit. The mass of the hydrogen is not being used efficiently!

If we take the radius to be that of a proton ($R = 10^{-13}$ cm.), then the amount of information that can be coded in the proton is only 44 bits! This is remarkably small considering the complexity of the proton — three valence quarks, innumerable sea quarks and gluons — so complex in fact that we have been unable to compute its ground state from first principles using the Standard Model even when we use our most advanced supercomputers. Bekenstein has used this extremely small number of possible states in the proton to constrain the number of possible quark fields that could be present in the quark sea.

In the early universe, where there are particle horizons, and also for black holes, the Bekenstein Bound in the form

$$I = \frac{S}{\ln 2} \leq \frac{A}{4L_P^2 \ln 2} = \frac{\pi R^2}{L_P^2 \ln 2} \quad (2.7)$$

is appropriate, where S is the total entropy in a causally connected region inside a 2-sphere of radius R and surface area A , where L_P is the Planck length. The Bekenstein Bound in the form (2.7) can be easily derived from (2.1) as follows.

If $R = 2GE/c^4$, then a black hole forms, enclosing the information, and *in asymptotically flat space* we cannot get any more energy into a sphere of radius R than this. Thus

$$I \leq \frac{2\pi ER}{\hbar c \ln 2} = 2\pi \left(\frac{Rc^4}{2G} \right) \frac{R}{\hbar c \ln 2} = 4\pi R^2 \left(\frac{c^3/G\hbar}{4 \ln 2} \right)$$

But $c^3/G\hbar = L_P^{-2}$ and $A = 4\pi R^2$, so we get (2.7). However, the formation of a black hole implies that there are event horizons, which means by definition that the final singularity cannot be an omega point. That is, inequality (2.7) applies if and only if the information corresponding to life is restricted to a part rather than the entire universe.

Bekenstein has noted [5] that when a region in the early universe with its particle horizons has a radius of the order of a Planck length L_P , the entropy and information must be of order one or less, from which he concludes that the initial singularity does not exist. I would instead interpret this result (which I believe to

be correct) as implying that the initial Friedmann singularity is *unique*; there is no information in the initial singularity. So $I = S = 0$ at the initial singularity, and thus there is no contradiction with the RHS of (2.7) going to zero as $R \rightarrow 0$. I shall show what this implies in section 3.

Ellis and Coule [6] argue that, in *any* closed universe near the *final* singularity, (2.7) is still the correct form of Bekenstein's Bound, with R being the scale factor of the universe, and thus $R \rightarrow 0$ means $I \rightarrow 0$, which obviously rules out $I \rightarrow +\infty$ as $R \rightarrow 0$, that holds if the Omega Point Theory is true. I shall show in Section H that if (2.1) rather than (2.7) is used, we can have $I \rightarrow +\infty$ as $R \rightarrow 0$, provided event horizons disappear.

But Ellis and Coule are wrong; (2.7) cannot be the correct form near the final singularity in a closed universe without event horizons because, if it were, then it would imply a global and universal violation of the Second Law of Thermodynamics when the radiation temperature reaches a mere 5×10^4 GeV, far below the Planck energy of 10^{19} GeV, and even far below the unification temperature where we think the Bekenstein Bound and the Second Law both apply.

To see this, write $S = SR^3$, where S is the entropy density, and let R_0 and T_0 be the scale factor and radiation temperature today. Using $R = R_0 T_0 / T$, (2.7) implies the following upper bound to the future universal temperature T :

$$T \leq \frac{\sqrt{\pi} T_0}{\sqrt{\mathcal{S}_0 R_0 L_p^2}} \quad (2.8)$$

We have $\mathcal{S}_0 = 2.9 \times 10^3 \text{ cm}^{-3}$ from equation (B.17) of Section B of [9]. (See also [7, p. 273]. Note that applying (B.17) to (2.8) requires leaving out the factor $\ln 2$.) Also, $T_0 = 2.726 \text{ }^\circ\text{K} = 2.349 \times 10^{-13} \text{ GeV}$. These numbers give

$$T \leq 5.3 \times 10^4 \text{ GeV} \quad (2.9)$$

if $R_0 = 3$ gigaparsecs (the Hubble distance) and $T \leq 3 \times 10^3 \text{ GeV}$ if $R_0 = 1$ teraparsec, the lower bound in Section B of [9]. If $R_0 = 10$ teraparsecs, the upper bound in Section B of [9], then $T \leq 1 \times 10^3 \text{ GeV}$, which is the energy the LHC is designed to probe. Surely quantum mechanics and the Second Law are valid at this energy, even in the collapsing phase of a closed universe. I shall show that this is in fact the case in Section 6.

b. Universe NOT Planck-sized at Planck Time

To show that the universe must have been much larger than the Planck Length at the Planck time, let us suppose the early universe was radiation dominated and topologically S^3 . Since we know that it would have had to be isotropic and homogeneous, it would be FRW and the scale factor $R(t)$ would evolve as

$$R(t) = R_{max} \left(\frac{2t}{R_{max}} - \frac{t^2}{R_{max}^2} \right)^{1/2}$$

where t is proper time, and R_{max} is the scale factor at maximum expansion. Putting in $t = L_{Pk}/c$ where L_{Pk} is the Planck length, and requiring $R(L_{Pk}/c) = L_{Pk}$ gives

$$R_{max} = L_{Pk}$$

That is, the universe's maximum size is the Planck length, in gross contradiction to observation.

If we assume that the CBR radiation has been present since the Planck time, which is to say that the universe's expansion has been adiabatic since the Planck time, then since in all cosmological models the radiation density $\rho \propto 1/R^4$, we have

$$R(L_{Pk}/c) = (\text{Hubble Distance}) \left(\frac{\rho_{today}}{\rho_{Pk}} \right)^{1/4} \approx 10^{-4} \text{ cm}$$

if we make the most natural assumption that the CBR had the Planck density ρ_{Pk} at the Planck time. I shall show how the universe attains its “unnatural” enormous size in Section 6.

c. FRW Universe does NOT admit a U(1) gauge field, like electromagnetism

It has been known by relativists for many years that a non-zero electromagnetic field — a U(1) YM field — cannot exist in a FRW universe. A simple proof is as follows. Equation (5.23) of Misner, Thorne and Wheeler ([8], p. 141) gives the stress energy tensor for the EM field in an orthonormal basis, in particular $T^{\hat{0}\hat{j}} = (\vec{E} \times \vec{B})^{\hat{j}}/4\pi$, which equals zero since in FRW there can be no momentum flow. Thus \vec{B} must be a multiple of \vec{E} , so set $\vec{B} = a\vec{E} = aE\hat{x}$. Computing the diagonal components of $T^{\mu\nu}$ gives $T^{\hat{0}\hat{0}} = E^2(1 + a^2)/8\pi \equiv \rho$, and $T^{\hat{x}\hat{x}} = -\rho = -T^{\hat{y}\hat{y}} = -T^{\hat{z}\hat{z}}$. But for FRW isotropy requires $T^{\hat{x}\hat{x}} = T^{\hat{y}\hat{y}} = T^{\hat{z}\hat{z}}$, so $\rho = (\vec{E}^2 + \vec{B}^2)/8\pi = 0$, which implies $\vec{E} = \vec{B} = 0$. However, any non-Abelian YM field with an SU(2) normal subgroup *can* be non-zero in a closed FRW, basically because SU(2) *is* a homogeneous and isotropic 3-sphere.

The fact that the FRW universe cannot admit an electromagnetic field is ignored in standard cosmology texts. What is done is to assume that the CMBR obeys the simple equation of state $p = \frac{1}{3}\rho$, and that the stress energy tensor is simply the perfect fluid stress energy tensor. The fact that the CMBR, if it indeed *is* electromagnetic radiation, is ignored, or more precisely, is assumed to result from some complicated averaging scheme which is never spelled out in detail. I shall argue throughout this paper that the most natural interpretation of this fact is that the CMBR is *not* a U(1) gauge field — it is *not* an electromagnetic field — and this extraordinary claim has experimental consequences, and amazingly, this extraordinary claim is consistent with all observations to day of the CMBR. I shall also argue that just as the CMBR was first seen in CN absorption lines, so the non-EM nature of the CMBR has actually already been seen in ultra high energy cosmic rays.

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3. The Spectra of Gauge Fields in a FRW Background

It has long been known (e.g, [6], p. 72; [7], p. 515) that the Rayleigh-Jeans long wavelength limit of the Planck distribution can be produced non-thermally, since this limit is simply an expression of the number of wave modes allowed in a given volume. However, I shall show that the spectral distribution of a quantized gauge Bose radiation field in an exact FRW cosmology must also necessarily follow a Planck distribution. More generally, any radiation field (defined as matter whose energy density is inversely proportional to the fourth power of the scale factor in a FRW cosmology) will necessarily obey the Wien displacement law, irrespective of whether it is quantized or what statistics the field obeys.

My derivation of the Planck distribution without assuming thermal equilibrium is analogous to Hawking's derivation of a Planckian distribution for the emission of radiation from a black hole. In Hawking's original calculation, no assumption of thermal equilibrium was made initially, but he discovered that the black hole radiation emission was Planckian, with the black hole surface gravity playing the role of the temperature. I shall show that in a FRW cosmology, a quantized gauge boson field must also have a Planck spectrum, with the quantity $\hbar c/R$, where R is the radius of the universe, playing the role of temperature. However, because of the isotropy and homogeneity of the FRW cosmology, there is no possibility of interpreting this quantity as a temperature.

a. PROOF THAT *ALL* CLASSICAL GAUGE FIELDS NECESSARILY OBEY A WIEN DISPLACEMENT LAW IN A FRW UNIVERSE

I shall first show that the spectral distribution of radiation — that is, any field whose energy density is inversely proportional to the fourth power of the radius of the universe — in any Friedmann-Robertson-Walker (FRW) cosmology necessarily obeys the Wien displacement law in the form

$$I(\lambda, R) = \frac{f(\lambda/R)}{R^5} = \frac{\phi(\lambda/R)}{\lambda^5} \quad (3.1)$$

where $R = R(t)$ is the FRW scale factor at any given time t , λ is the wavelength, $I(\lambda, R)$ is the monochromatic emissive power of the radiation, and $f(\lambda/R)$ and $\phi(\lambda/R)$ are unknown functions of the single variable λ/R . Notice that in the form (3.1), the reciprocal of the scale factor has replaced the temperature T . The temperature does not appear in the form (3.1), and no assumption of thermodynamic equilibrium will be used in the derivation of (3.1). That is, the spectral distribution (3.1) will apply no matter what the thermodynamic state of the radiation is; it will even be consistent with the radiation being at absolute zero.

Recall that in the standard derivation of the Wien displacement law, the first step is to establish Kirchhoff's law, which yields the fact that the intensity I of the radiation at a given wavelength λ depends only on λ and the absolute temperature. In the standard derivation of Kirchhoff's law, the assumption of thermal equilibrium is required to establish this. If we have a single radiation field in a FRW cosmology, then $I = I(\lambda, R)$ — the intensity at a given wavelength depends only on the wavelength and the scale factor — because there are no other variables in the cosmology upon which the intensity of radiation could depend.

Second, we recall that in a closed universe, the number of modes \mathcal{N} is constant under the expansion:

$$\mathcal{N} = \frac{R}{\lambda} = \frac{R'}{\lambda'} \quad (3.2a)$$

where the primes denote the quantities at some other time. Equation 3.2a) can be re-written

$$\frac{R'}{R} = \frac{\lambda'}{\lambda} \quad (3.2b)$$

An alternative calculation following [1], pp. 777–778 shows that in addition the same relation between the wavelengths and expansion factors also hold infinitesimally: $d\lambda/R = d\lambda'/R'$, or

$$\frac{d\lambda'}{d\lambda} = \frac{R'}{R} \quad (3.2c)$$

During the expansion, the energy density U of a radiation dominated universe also changes. We have

$$dU = \left(\frac{4}{c}\right) I(\lambda, R) d\lambda \quad (3.3)$$

The energy density of any gauge field satisfies

$$\frac{dU}{dU'} = \left(\frac{R'}{R}\right)^4 \quad (3.4)$$

Thus combining (3.3) and (3.4) gives

$$\frac{dU}{dU'} = \frac{\left(\frac{4}{c}\right) I(\lambda, R) d\lambda}{\left(\frac{4}{c}\right) I(\lambda', R') d\lambda'} = \left(\frac{R'}{R}\right)^4 \quad (3.5)$$

which gives upon solving for $I(\lambda, R)$ while using (3.2b) and (3.2c):

$$I(\lambda, R) = \left(\frac{R'}{R}\right)^4 \frac{d\lambda'}{d\lambda} I(\lambda', R') = \left(\frac{R'}{R}\right)^5 I\left(\frac{\lambda R'}{R}, R'\right) = \left(\frac{\lambda'}{\lambda}\right)^5 I\left(\frac{\lambda R'}{R}, R'\right) \quad (3.6)$$

As in the usual derivation of the Wien displacement law, we note that since the LHS of equation (3.6) does not contain the variables R' or λ' , neither can the RHS. Thus (3.6) can be written

$$I(\lambda, R) = \frac{f(\lambda/R)}{R^5} = \frac{\phi(\lambda/R)}{\lambda^5} \quad (3.1)$$

which is the Wien displacement law. Notice that if there were several non-interacting radiation fields present, then each would satisfy the Wien displacement law, but possibly with different functions of R , since we are not assuming thermal equilibrium.

The maximum of the distribution (3.1) is obtained by differentiating the first form of (3.1) with respect to λ :

$$\left. \frac{dI(\lambda, R)}{d\lambda} \right|_{\lambda=\lambda_m} = \left. \frac{d}{d\lambda} \frac{f(\lambda/R)}{R^5} \right|_{\lambda=\lambda_m} = \frac{1}{R^6} f'(\lambda_m/R) = 0$$

which tells us that the wavelength λ_m of the maximum of the distribution satisfies

$$\frac{\lambda_m}{R} = \text{constant} \quad (3.7)$$

Of course, the above calculation is meaningful only if there *is* a maximum. The Rayleigh-Jeans law obeys the Wien displacement law, and has no maximum. But the Rayleigh-Jeans law also suffers from the ultraviolet divergence, and so is unphysical. The actual distribution $I(\lambda, R)$ must either have a maximum, or asymptotically approach a limiting value as $\lambda \rightarrow 0$.

b. PROOF THAT *ALL* QUANTIZED GAUGE FIELDS NECESSARILY HAVE A PLANCKIAN SPECTRUM IN A FRW UNIVERSE

I shall now show that if the radiation field of Section 1 is in addition a quantized gauge boson gas, the spectral distribution will follow the Planck distribution irrespective of whether the gas is in thermal equilibrium. The key idea will be to follow Planck's original derivation of his Law [2, 3], which remarkably did NOT assume that the gas was at a maximum of the entropy (i.e., did not assume equilibrium), though, as is well-known, he did assume in effect assume the energies of the gas particles were quantized, and that these particles obeyed Bose statistics. As in the derivation of the Wien displacement law, the reciprocal of the scale factor of the FRW cosmology will replace the temperature in the Planck distribution law.

The first part of the derivation will be the same as the standard derivation of the Planck distribution.

Let us define the following quantities:

$g_s \equiv$ number of modes in the s energy level;

$n_s \equiv$ number of particles in the s energy level;

$\epsilon \equiv$ the energy of the s energy level; and

$Q \equiv n_s/g_s =$ the occupation index.

For a boson gas, the number of distinct arrangements is

$$P_s = \frac{(n_s + g_s - 1)!}{n_s!(g_s - 1)!} \quad (3.8)$$

The number of distinct arrangements P for all energy levels is thus

$$P = \prod_{s=1}^{s_{max}} P_s = \prod_{s=1}^{s_{max}} \frac{(n_s + g_s - 1)!}{n_s!(g_s - 1)!} \quad (3.9)$$

The information in this collection of possible arrangements is

$$I \equiv \log P = \sum_{s=1}^{s_{max}} [\log(n_s + g_s - 1)! - \log n_s! - \log(g_s - 1)!] \quad (3.10)$$

If we assume $g_s \gg 1$, and use Stirling's formula, (3.10) becomes

$$\begin{aligned} I &= \sum_{s=1}^{s_{max}} [(n_s + g_s) \log(n_s + g_s) - n_s \log n_s - g_s \log g_s] = \\ I &= \sum_{s=1}^{s_{max}} g_s \left[\left(1 + \frac{n_s}{g_s}\right) \log \left(1 + \frac{n_s}{g_s}\right) - \frac{n_s}{g_s} \log \frac{n_s}{g_s} \right] \end{aligned} \quad (3.11)$$

where each term in the sum will be denoted I_s , the information in each energy level.

Now we know by the Bekenstein Bound that

$$I \leq \text{constant}(RE)$$

In the situation of perfect isotropy and homogeneity this must apply for each state s independently, and for each mode. Since the information per mode can depend only on the scale factor R — in the FRW universe there is no other possible function for the information per mode to depend on — and since the Bekenstein Bound gives a linear dependence on R for all values of the scale factor, the inequality can be replaced by an equality:

$$d(I_s/g_s) = \mathcal{T} R \epsilon_s d(n_s/g_s) = \frac{\partial(I_s/g_s)}{\partial(n_s/g_s)} d(n_s/g_s) \quad (3.12)$$

where \mathcal{T} is a constant to be determined. Equation (3.12) can be written

$$\frac{\partial(I_s/g_s)}{\partial(n_s/g_s)} = \mathcal{T}\epsilon_s R \quad (3.13)$$

From equation (3.11) we have

$$\frac{I_s}{g_s} = \left(1 + \frac{n_s}{g_s}\right) \log\left(1 + \frac{n_s}{g_s}\right) - \frac{n_s}{g_s} \log \frac{n_s}{g_s}$$

and so substituting for simplicity $Q \equiv n_s/g_s$ we can evaluate the partial derivative in (3.13):

$$\frac{\partial(I_s/g_s)}{\partial(n_s/g_s)} = \frac{d}{dQ} [(1+Q) \log(1+Q) - Q \log Q] = \log\left(\frac{1+Q}{Q}\right) = \epsilon_s \mathcal{T} R$$

which can be solved for

$$n_s = \frac{g_s}{\exp(\epsilon_s \mathcal{T} R) - 1} \quad (3.14)$$

As is well-known, the infinitesimal number of modes dN in a volume V in the frequency interval $d\omega$ is

$$dN = \frac{V\omega^2 d\omega}{\pi^2 c^3}$$

so the energy per mode is

$$dE = \hbar\omega n_s/g_s = \hbar\omega dN/g_s = \frac{\hbar\omega V\omega^2 d\omega}{\pi^2 c^3 (\exp(\hbar\omega \mathcal{T} R) - 1)}$$

which yields a spectral energy density $dU = dE/V$ of

$$dU = \frac{\hbar\omega^3 d\omega}{\pi^2 c^3 (\exp(\hbar\omega \mathcal{T} R) - 1)} \quad (3.15)$$

Using $I(\lambda, R) = (4/c)dU$ we obtain

$$I(\lambda, R) = \frac{2\pi c^2 h}{\lambda^5 (\exp(\mathcal{T} c h R/\lambda) - 1)} \quad (3.16)$$

Equation (3.15) can be integrated over all frequencies from 0 to $+\infty$ to give at total energy density

$$U = \frac{\pi^2}{15\hbar^3 c^3} \left(\frac{1}{\mathcal{T} R}\right)^4 \quad (3.17)$$

In a radiation dominated closed FRW universe, we have (e.g. [1], p. 735)

$$U = \frac{3R_{max}^2 c^4}{8\pi G R^4} \quad (3.18)$$

where R_{max} is the scale factor at maximum expansion.

Equating (3.17) and 3.18) yields the constant \mathcal{T} :

$$\mathcal{T} = \left(\frac{8\pi^3}{45}\right)^{1/4} \left(\frac{L_{Pk}}{R_{max}}\right)^{1/2} \left(\frac{1}{\hbar c}\right) \quad (3.19)$$

If we integrate equation (3.16) over all λ to obtain the total information in the universe, we obtain

$$I_{Total} = \frac{2\pi^4}{15} \left(\frac{45}{8\pi^3}\right)^{3/4} \left[\frac{R_{max}}{L_{Pk}}\right]^{3/2} \approx 4 \left[\frac{R_{max}}{L_{Pk}}\right]^{3/2} \quad (3.20)$$

This is independent of the scale factor of the universe R — so the information in the gauge field does not change with time (a Planck distribution is unchanged by the universal expansion), but nevertheless it

is non-zero, which may be contrary to expectation; one might expect that the information in the universe is zero in the absence of thermalization.

Before addressing the origin of this information (hint: the origin is obvious from equation (3.20)), let me first point out that the number (3.20) is completely consistent with the Bekenstein Bound, which is

$$I \leq \left(\frac{2\pi}{\hbar c} \right) (ER) \quad (3.21)$$

Let me replace the constant $2\pi/\hbar c$ with the constant \mathcal{T} , which I have assumed will give equality:

$$I = \mathcal{T} (ER) \quad (3.22)$$

where \mathcal{T} can be written

$$\mathcal{T} = \frac{1}{(90\pi)^{1/4}} \left(\frac{L_{Pk}}{R_{max}} \right)^{1/2} \left(\frac{2\pi}{\hbar c} \right) \quad (3.23)$$

So, provided

$$R_{max} \geq L_{Pk} \quad (3.24)$$

we will have

$$\mathcal{T} < \frac{2\pi}{\hbar c} \quad (3.25)$$

and thus the Bekenstein Bound will hold.

If we happened to be in a universe in which $R_{max} < L_{Pk}$, then the crucial approximation $g_s \gg 1$ and the use of Stirling's formula, which allowed me to re-write (3.10) as (3.11), would no longer be valid, and we would obtain a different \mathcal{T} , consistent with the Bekenstein Bound in this universe.

Thus the origin of the information in the universe is the particular value for R_{max} , which is just one of the possible values; as we shall see in Section 6, there are an infinity of possible values, and our "selection" of the particular scale factor at maximum expansion in our particular universe of the "multiverse" (a term which will be precisely defined in Section 6), generates the information.

In fact, it is clear from (3.23) that had we simply imposed the requirement that the information be of order 1 at the Planck radius, say by setting $\mathcal{T} = 1$, then $R_{max} \sim L_{Pk}$. Alternatively, let us try to eliminate all reference to R_{max} , by the dimensionally allowed replacement $kT \rightarrow \hbar c/R$. Then, using the standard expression above for the energy density U of radiation with temperature T , we get

$$U = \frac{\pi^2 (kT)^4}{15(\hbar c)^3} = \frac{\pi^2}{15(\hbar c)^3} \left(\frac{\hbar c}{R} \right)^4 = \frac{\pi^2 \hbar c}{15R^4} = \frac{3c^4}{8\pi G R_{max}^2 \sin^4 \tau} = \frac{3R_{max}^2 c^4}{8\pi G R^4}$$

or,

$$\frac{3R_{max}^2}{8\pi G} = \frac{\pi^2 \hbar c}{15}$$

which yields

$$R_{max} = \frac{2\pi}{\sqrt{15}} L_{Pk} \approx (1.6) L_{Pk} \quad (3.26)$$

Which is to say, the scale factor at maximum expansion is of the order of the Planck length.

Another way we could try to set the constant \mathcal{T} is to simply require that the information in the gauge field be of the order of one bit. (Since the expansion is adiabatic, the radius will cancel out of the calculation.) Recall that the entropy of radiation is given by

$$S = \frac{4\pi^2 k V (kT)^3}{45 (c\hbar)^3} \quad (3.27)$$

Setting $kT = 1/\mathcal{T}R$ and using the volume of a closed universe $V = 2\pi^2 R^3$, we get

$$\mathcal{T} = \frac{2\pi}{\hbar c} \left(\frac{\pi}{45}\right)^{1/3} \left(\frac{k}{S}\right)^{1/3} \quad (3.28)$$

Setting $S/k = 1$ gives

$$\mathcal{T} = \frac{2\pi}{\hbar c} \left(\frac{\pi}{45}\right)^{1/3} \approx \frac{2.6}{\hbar c} \quad (3.29)$$

which, as we have already seen, gives $R_{max} \approx L_{Pk}$.

Another alternative we could try would be to set the number of quanta in the universe to be equal to one. Recall that the number n of quanta is

$$n = \frac{2\zeta(3)}{\pi^2 c^3 \hbar^3} (kT)^3 V = \frac{2\zeta(3)}{\pi^2 c^3 \hbar^3} \left(\frac{1}{\mathcal{T}R}\right) (2\pi^2 R^3) = \frac{4\zeta(3)}{(c\hbar\mathcal{T})^3}$$

Setting $n = 1$ gives

$$\mathcal{T} c \hbar = (4\zeta(3))^{1/3} \approx 1.7$$

which once again gives $R_{max} \approx L_{Pk}$.

As Bekenstein has often pointed out, when horizons are present, the correct “size” R that really should be put into the Bekenstein Bound is the horizon radius; in the case of the early universe, this is the particle horizon radius. Let us show now that with this value for “size”, indeed the choice (3.19) gives less than one bit of information inside the horizon when the particle horizon is the Planck Length.

The equation for the particle horizon radius is

$$ds^2 = -dt^2 + R^2(t)d\chi^2 \equiv -dt^2 + (dR_{Particle})^2 = 0$$

which when integrated (setting $R(0) = 0$) yields

$$R_{Particle} = t \quad (3.30)$$

It is well known that for a radiation dominated FRW closed universe the scale factor can be expressed in closed form in terms of the proper time t :

$$R(t) = R_{max} \left(\frac{2t}{R_{max}} - \frac{t^2}{R_{max}^2} \right)^{1/2} \quad (3.31)$$

which can be solved for the proper time t :

$$t = R_{max} \left(1 - \sqrt{1 - \left(\frac{R}{R_{max}} \right)^2} \right) \quad (3.32)$$

valid for $0 < t \leq R_{max}$. For $R \ll R_{max}$, this is approximately

$$R_{Particle} \approx \frac{R^2(t)}{2R_{max}} \quad (3.33)$$

The information inside the particle horizon in the early universe is thus

$$I = \mathcal{T} (U R_{Particle}^3) (R_{Particle}) = \mathcal{T} U R_{Particle}^4 = \mathcal{T} \left(\frac{3R_{max}^2 c^4}{8\pi G R^4} \right) \left(\frac{R^8}{2R_{max}^4} \right) =$$

$$= \left(\frac{8\pi^3}{45}\right)^{1/4} \left(\frac{L_{Pk}}{R_{max}}\right)^{1/2} \left(\frac{3R^4}{8\pi R_{max}^2 L_{Pk}^2}\right) \quad (3.34)$$

Putting (3.33) into (3.34) gives

$$I = \left(\frac{8\pi^3}{45}\right)^{1/4} \left(\frac{L_{Pk}}{R_{max}}\right)^{1/2} \left(\frac{12R_{particle}^2}{L_{Pk}^2}\right) \quad (3.35)$$

which is much, much less than one for $R_{particle} \leq L_{Pk}$.

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4. Particle Production Solution to the EYM Equation in a FRW Universe

Exact Solution of EYM Equations with Constant SU(2) Curvature

The Yang-Mills field (curvature) is

$$W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu + g f_{abc} W_b^\mu W_c^\nu$$

where f_{abc} are the structure constants of the Lie group defining the Yang-Mills field, the Latin index is the group index, and we summation over all repeated Latin indices. In the absence of all other fields except gravity, the YM fields satisfy the equations [10, p. 13]

$$\nabla \wedge W = 0$$

and

$$\nabla \wedge^* W = 0$$

where ∇ is the gauge and spacetime covariant derivative. The first equation is the Bianchi identity for the YM fields, while the second is the Yang-Mills equation. It is obvious that a self-dual ($W =^* W$) or anti-self-dual ($W = -^* W$) will automatically satisfy both equations if it satisfies one.

In more conventional notation, the Bianchi identity is

$$D_\mu W_{\nu\lambda}^a + D_\nu W_{\lambda\mu}^a + D_\lambda W_{\mu\nu}^a = 0$$

where

$$D_\lambda W_{\mu\nu}^a = W_{\mu\nu;\lambda}^a - f_{bc}^a A_\lambda^c W_{\mu\nu}^b$$

with the semicolon denoting the spacetime covariant derivative and A_λ^c being the gauge potential, in terms of which the gauge field $W_{\mu\nu}^a$ can be expressed as

$$\begin{aligned} W_{\mu\nu}^a &= A_{\nu;\mu}^a - A_{\mu;\nu}^a + f_{bc}^a A_\mu^b A_\nu^c \\ &= A_{\nu,\mu}^a - A_{\mu,\nu}^a + f_{bc}^a A_\mu^b A_\nu^c \end{aligned}$$

where the last equality is valid in any coordinate basis; that is, if the spacetime covariant derivative is expressed in a coordinate basis, the spacetime covariant derivatives can be replaced by partial derivatives. The same is true in the Bianchi identity for the gauge fields W .

The Yang-Mills equation in more conventional notation is [11, p. 12]:

$$D^\nu W_{\mu\nu}^a = 0 = W_{\mu}^a{}^\nu{}_{;\nu} - f_{bc}^a A_\nu^c W_{\mu}^b{}^\nu$$

The Lagrangian for the YM field is $L = -(1/16\pi)W_a^{\mu\nu}W_{\mu\nu}^a$, and the standard expression for the stress energy tensor $T_{\mu\nu} = -2\delta L/\delta g^{\mu\nu} + g_{\mu\nu}L$ yields

$$T_{\mu\nu}^{YM} = \frac{1}{4\pi} \left[W_{\mu\beta}^a W_{a\nu}^\beta - \frac{1}{4} g_{\mu\nu} W_{\alpha\beta}^a W_a^{\alpha\beta} \right]$$

For any $T_{\mu\nu}^{YM}$, we have $T_\mu^\mu = 0$, and so for an isotropic and homogeneous universe, any YM field must have $T_{\hat{i}\hat{i}} \equiv p = \frac{1}{3}T_{\hat{t}\hat{t}}$, where $T_{\hat{t}\hat{t}} \equiv \rho$ in any local orthonormal frame. In other words, any YM field satisfies in a FRW universe a perfect fluid equation of state with adiabatic index $\gamma = 4/3$.

However, very few Yang-Mills fields are consistent with isotropy and homogeneity. It is well-known that a non-zero electromagnetic field — a U(1) YM field — cannot exist in a FRW universe. (Proof: eq. (5.23) of [1], p. 141, gives the stress energy tensor for the EM field in an orthonormal basis, in particular $T^{\hat{0}\hat{j}} = (\vec{E} \times \vec{B})^{\hat{j}}/4\pi$, which equals zero since in FRW there can be no momentum flow. Thus \vec{B} must be a multiple of \vec{E} , so set $\vec{B} = a\vec{E} = aE\hat{x}$. Computing the diagonal components of $T^{\mu\nu}$ gives $T^{\hat{0}\hat{0}} = E^2(1 + a^2)/8\pi \equiv \rho$, and $T^{\hat{x}\hat{x}} = -\rho = -T^{\hat{y}\hat{y}} = -T^{\hat{z}\hat{z}}$. But for FRW isotropy requires $T^{\hat{x}\hat{x}} = T^{\hat{y}\hat{y}} = T^{\hat{z}\hat{z}}$, so $\rho = (\vec{E}^2 + \vec{B}^2)/8\pi = 0$, which implies $\vec{E} = \vec{B} = 0$). However, any non-Abelian YM field with an SU(2) normal subgroup *can* be non-zero in a closed FRW, basically because SU(2) is a homogeneous and isotropic 3-sphere.

If the YM connection is a left invariant 1-form, that is, if the connection is a Maurer-Cartan form, then the resulting YM curvature will be given spatially by the structure constants of the Lie group. The YM curvature will be

$$W_a^{\mu\nu} = g f_{abc} W_b^\mu W_c^\nu$$

where

$$W_a^\mu = R^{-1}(t) \delta_a^\mu$$

with $a = 1, 2, 3$ being the spatial indices and $R(t)$ being the FRW scale factor. It is easily checked that the above expression for $T_{\mu\nu}^{YM}$ gives $T^{\hat{j}\hat{j}} = (1/3)T^{\hat{0}\hat{0}} \propto R^{-4}$ and all other components zero, provided that $f_{abc} = \epsilon_{abc}$, the structure constants for SU(2).

It is clear that the above expression for the gauge field is consistent only if $R(t) = \text{constant}$. The true time dependent SU(2) gauge field is

$$W_a^{\mu\nu} = \left[\epsilon_{abc} \delta_b^\mu \delta_c^\nu \pm \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \epsilon_{abc} \delta_\alpha^b \delta_\beta^c \right] \frac{A}{R^2(t)}$$

where A is a constant, fixed by the Higgs field as I shall show below. The plus sign gives a self-dual gauge field ($W_a^{\mu\nu} = +^*W_a^{\mu\nu}$), and the minus sign gives an anti-self-dual field ($W_a^{\mu\nu} = -^*W_a^{\mu\nu}$).

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5. Particle Production by Instanton Tunnelling in a FRW Universe

Since the Bekenstein Bound requires a unique initial state, and since the only allowed non-zero initial field is the isotropic and homogeneous $SU(2)$ field, the initial baryon number is necessarily zero; the Bekenstein Bound thus *requires* a mechanism to produce a net baryon number. Baryogenesis requires satisfying the three Sakharov conditions:

- (1) violation of baryon number B and lepton number L conservation;
- (2) violation of C and CP invariance; and
- (3) absence of thermal equilibrium

The SM has a natural method of baryogenesis via the triangle anomaly, which generates both baryon and lepton number ($B + L$ is not conserved but $B - L$ is conserved), and since the self-dual gauge field generates fermions rather than anti-fermions, it violates C . The anomaly function $*W_{\mu\nu}^a W_a^{\mu\nu}$ can be written $\mathbf{E}^a \cdot \mathbf{B}_a$. Since \mathbf{E}^a is odd under parity while \mathbf{B}_a is even, the anomaly function will violate CP . At zero temperature, all processes will be effectively non-equilibrium process. So baryogenesis via the triangle anomaly at zero temperature is the natural method of SM baryogenesis.

The Standard Model violates CP perturbatively via the complex phase in CKM matrix. In the early universe, this perturbative mechanism fixes whether fermions or anti-fermions will be created via the triangle anomaly; that is, it fixes the $SU(2)$ gravitational sphaleron to be a self-dual rather than an anti-self-dual solution to the EYM equations. At high temperatures, the triangle anomaly will on average generate almost as many anti-fermions as fermions, because in thermal equilibrium the $SU(2)$ gauge fields will be anti-self-dual as often as self-dual. Thus, in equilibrium, the CP violating CKM complex phase acts to suppress SM baryogenesis; the excess of fermions over anti-fermions is suppressed by the Jarlskog determinant factor. As is well-known, this suppression can wash out at 100 GeV any fermion excess generated at higher temperature by other mechanisms.

In the usual high temperature SM baryogenesis calculation ([1], [2], [3], the baryon to photon ratio in dimensionless units, known (e.g. [4]) from nucleosynthesis to be $\eta_{-9} = 1.0 \pm 0.15$, is too small by a factor of about 10^{-8} , because the net creation rate is suppressed by the smallness of the CP violation in the CKM matrix described above, even when the problem of washing out any net baryon number is ignored. These problems are avoided in my proposed mechanism, for two reasons: first, the only role played by the CP violation is to select a self-dual rather than an anti-self dual field as the unique field (in the absence of CP violation there would be no unique $SU(2)$ field; the actual magnitude of the CP violation is irrelevant. Second, the baryon number generation is always at zero temperature, so there will never be any anti-fermions generated, and never any washout of fermions created. In fact, my model may have the opposite problem from the usual electroweak high temperature model: my model may tend to produce too many baryons relative to the number of $SU(2)$ pseudo-photons..

The net baryon number generated by the $SU(2)$ sphaleron is given by [2; 5, p. 454]:

$$N = \frac{-1}{32\pi^2} \int \sqrt{-g} d^4x \left[\frac{1}{2} \epsilon^{\alpha\beta\mu\nu} W^a_{\alpha\beta} W^b_{\mu\nu} (tr t_a t_b) \right] \quad (5.1)$$

I have set the $SU(3)$ gauge field to zero. Once again, this is required by uniqueness. There are uncountably many $SU(2)$ subgroups in $SU(3)$, but they are all in the same conjugacy class. A simple proof of this is as follows (this simple proof was pointed out to me by J. Bryan).

Suppose G is a subgroup of $SU(3)$ and G is isomorphic to $SU(2)$. Then the action of $SU(3)$ on C^3 (complex Euclidean 3-space) induces a three dimensional representation of G . Since any representation is the direct sum of irreducible representations, this representation must be (1) the unique irreducible representation

of dimension three (spin one representation), or (2) a direct sum of the unique representation of dimension two (spin one half representation) plus the one dimensional (trivial, spin zero) representation, or (3) a direct sum of three one dimensional (trivial) representations. (I use the fact that $SU(2)$ has a unique irreducible representation in each dimension). It cannot be (3) since G is a subgroup and so acts non-trivially. It cannot be (1) since this representation is isomorphic to the adjoint representation of $SU(2)$ on its Lie algebra and so the negative of the identity element acts trivially and so G would not be a subgroup. Therefore the representation must be a direct sum of the standard two dimensional representation and the trivial representation. Choose a unitary basis of C^3 so that the last factor is the trivial representation and first two factors are the standard representation and this change of basis will conjugate G into the standard embedding of $SU(2)$ into $SU(3)$. QED. (As an aside, note that we have the double cover $SU(2) \rightarrow SO(3) \subset SU(3)$. The induced representation on $SU(2)$ in this case will in fact be the irreducible three dimensional one, but in this case the subgroup is $SO(3)$, not $SU(2)$.)

However, even though all $SU(2)$ subgroups are in the same conjugacy class, they are not physically equivalent. Each different $SU(2)$ subgroup is generated by a different Lie subalgebra corresponding to a different linear superposition of gluons. Each such linear superposition is physically different. Thus there are uncountably many physically distinct $SU(2)$ subgroups of $SU(3)$, each capable of generating a homogeneous and isotropic metric (since isomorphic to the electroweak $SU(2)$ used above). This means the only way to have a unique $SU(3)$ field over a FRW spacetime is to set *all* the gluons fields to zero.

I have exhibited the two essentially unique vacuum solutions to the ETM equations in Section 4; since, as I have argued above, the self-dual is the unique solution required by the SM, we take the plus sign:

$$W_a^{\mu\nu} = \left[\epsilon_{abc} \delta_b^\mu \delta_c^\nu \pm \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \epsilon_{abc} \delta_\alpha^b \delta_\beta^c \right] \frac{A}{R^2(t)} \quad (5.2)$$

Putting (5.2) into (5.1) gives

$$N = \frac{1}{32\pi^2} \int \frac{6A^2}{R^4} \sqrt{-g} d^4x = \frac{3\pi}{8} A^2 \int_0^t \frac{dt}{R} \quad (5.3)$$

The last integral is of course conformal time; net fermion production is proportional to conformal time, and this suggests that the most appropriate time variable for quantum gravity is conformal time, since conformal time and only conformal time measures the rate at which something new is occurring to break perfect symmetry: fermions are appearing in the pure $SU(2)$ gauge field. This fact of the nature of the natural time variable will be used in Section 6 to quantize the gravitational field in the early universe.

The reader should be aware that I have not shown that my mechanism will in fact produce the correct observed baryon to photon ratio; I have only argued that my mechanism is not in principle subject to the well-known limitations of SM electroweak baryogenesis. I conjecture that my mechanism will produce the correct ratio; investigation of my conjecture will be subject of a subsequent paper.

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6. The Unique Quantized FRW Universe

a. Conformal Time is the Unique Physical Time

In this section, I shall justify ignoring quantum gravity effects — quantum gravity fluctuations — in the very early universe. I shall do this by constructing a quantized FRW universe in which the only field is a gauge field (actually a perfect fluid for which $p = \rho/3$) and show that imposing the boundary condition that classical physics hold exactly at “late times” (any time after the first minute) implies that classical physics is good all the way into the initial singularity.

In standard quantum gravity, the wave function of the universe obeys the Wheeler-DeWitt equation

$$\hat{\mathcal{H}}\Psi = 0 \quad (6.1)$$

where $\hat{\mathcal{H}}$ is the super-Hamiltonian operator. This operator contains the equivalent of time derivatives in the Schrödinger equation. I say “the equivalent” because quantum gravity does not contain time as an independent variable. Rather, other variables — matter or the spatial metric — are used as time markers. In other words, the variation of the physical quantities *is* time. Depending on the variable chosen to measure time, the time interval between the present and the initial or final singularity can be finite or infinite — but this is already familiar from classical general relativity. In the very early universe, conformal time measures the rate at which particles are being created by instanton tunnelling, that is it measures the rate at which new information is being created. Therefore, the most appropriate physical time variable is conformal time, and thus we shall select an appropriate combination of matter and spatial variables that will in effect result in conformal time being used as the fundamental time parameter in the Wheeler-DeWitt equation. Conformal time is also the most natural physical time to use for another reason: The matter in the early universe consists entirely of an $SU(2)$ gauge field, and the Yang-Mills equation is conformally invariant; a gauge field’s most natural time variable is conformal time.

Since the Bekenstein Bound tells us that the information content of the early universe is zero, this means that the only physical variable we have to take into account is the scale factor R of the universe, and the density and pressure of the gauge field. So we only have to quantize the FRW universe for a radiation field, or equivalently, a perfect fluid for which $p = \rho/3$.

If matter is in the form of a perfect fluid, the action S in the ADM formalism can be written

$$S = \int (\mathfrak{R} + p)\sqrt{-g} d^4x = \int L_{ADM} dt \quad (6.2)$$

where p is the fluid pressure and \mathfrak{R} is the Ricci scalar. If the spacetime is assumed to be a Friedmann universe containing isentropic perfect fluids, Lapchinskii and Rubakov [2] have shown the canonical variables can be chosen (R, ϕ, s) , where R is the scale factor of the universe, and ϕ, s are particular parameterizations of the fluid variables called *Schutz potentials* [3]. The momenta conjugate to these canonical variables will be written (p_R, p_ϕ, p_s) .

The ADM Lagrangian in these variables can be shown to be

$$L_{ADM} = p_R R' + p_\phi \phi' + p_s s' - N(H_g + H_m) \quad (6.3)$$

where the prime denotes the time derivative,

$$H_g = -\frac{p_R^2}{24R} - 6R \quad (6.4)$$

is the purely gravitational super-Hamiltonian, and

$$H_m = N^2 R^3 [(\rho + p)(u^0)^2 + p g^{00}] = p_\phi^\gamma R^{3(1-\gamma)} e^s \quad (6.5)$$

is both the coordinate energy density measured by a comoving observer and the super-Hamiltonian of the matter. The momentum conjugate to R , the scale factor of the universe, is

$$p_R = -\frac{12RR'}{N} \quad (6.6)$$

The constraint equation for the Friedmann universe is obtained by substituting (6.3) – (6.5) into (6.2) and varying the lapse N . The result is the super-Hamiltonian constraint:

$$0 = \mathcal{H} = H_g + H_m = -\frac{p_R^2}{24R} - 6R + p_\phi^\gamma R^{3(1-\gamma)} e^s \quad (6.7)$$

When the perfect fluid is radiation the last term is $H_m = p_\phi^{4/3} e^s / R$, and so if we choose the momentum conjugate to the *true* time τ to be

$$p_\tau = p_\phi^{4/3} e^s \quad (6.8)$$

then the super-Hamiltonian constraint becomes

$$0 = \mathcal{H} = -\frac{p_R^2}{24R} - 6R + \frac{p_\tau}{R} \quad (6.9)$$

The ADM Hamiltonian is obtained from $H_{ADM} = p_\tau$, or

$$H_{ADM} = \frac{p_R^2}{24} + 6R^2 \quad (6.10)$$

which is just the Hamiltonian for a simple harmonic oscillator.

The lapse N is fixed by solving Hamilton's equation

$$\tau' = 1 = \frac{\partial(N[H_g + H_m])}{\partial p_\tau} = \frac{N}{R} \quad (6.11)$$

which says that $N = R$; that is, *true* time is just conformal time, which is why I have called it τ .

If we quantize by the replacement $p_\tau \rightarrow \hat{p}_\tau = -i\partial/\partial\tau$, and $p_R \rightarrow \hat{p}_R = -i\partial/\partial R$, together with a reversal of the direction of time $\tau \rightarrow -\tau$ in the super-Hamiltonian constraint (6.9), the Wheeler-DeWitt equation (6.1) will then become (if we ignore factor ordering problems) Schrödinger's equation for the simple harmonic oscillator with mass $m = 12$, spring constant $k = 12$ and angular frequency $\omega = 1$:

$$i\frac{\partial\Psi}{\partial\tau} = -\frac{1}{24}\frac{\partial^2\Psi}{\partial R^2} + 6R^2\Psi \quad (6.12)$$

b. Consistency between Copenhagen and Many-Worlds Interpretations Requires a Delta Function Initial Boundary Condition

We need to find what boundary conditions to impose on equation (6.12). The boundary condition that I propose is the unique boundary condition that will allow the classical Einstein equations to hold exactly in the present epoch: that is, I shall require that on the largest possible scales in the present epoch, classical mechanics holds exactly. To see how to impose such a boundary condition, let us consider the general one-particle Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{x})\psi \quad (6.13)$$

If we substitute ([4], p. 280; [6], p. 51–52; [7])

$$\psi = \mathcal{R}\exp(i\varphi/\hbar) \quad (6.14)$$

into (6.13), where the functions $\mathcal{R} = \mathcal{R}(\vec{x}, t)$ and $\varphi = \varphi(\vec{x}, t)$ are real, we obtain

$$\frac{\partial \mathcal{R}}{\partial t} = -\frac{1}{2m} \left[\mathcal{R} \nabla^2 \varphi + 2 \vec{\nabla} \mathcal{R} \cdot \vec{\nabla} \varphi \right] \quad (6.15)$$

$$\frac{\partial \varphi}{\partial t} = -\frac{(\vec{\nabla} \varphi)^2}{2m} - V + \left(\frac{\hbar^2}{2m} \right) \frac{\nabla^2 \mathcal{R}}{\mathcal{R}} \quad (6.16)$$

Equation (6.16) is just the classical Hamilton-Jacobi equation for a single particle moving in the potential

$$U = V - \left(\frac{\hbar^2}{2m} \right) \frac{\nabla^2 \mathcal{R}}{\mathcal{R}} \quad (6.17)$$

Equations (6.16) and (6.17) are fully equivalent to Schrödinger's equation (6.13), and this way of expressing Schrödinger's equation, called the Bohm–Landau Picture ([6], [7]), is the most convenient formulation of QM when one wishes to compare QM with classical mechanics. The normals to surfaces of constant phase, given by $\varphi(\vec{x}, t) = \text{constant}$, define trajectories: those curves with tangents

$$\vec{\nabla} \varphi = \frac{\hbar}{2im} \ln \left(\frac{\psi}{\psi^*} \right) = \text{Re} \left[\left(\frac{\hbar}{i} \right) \ln \psi \right] \quad (6.18)$$

The density of the trajectories is conserved, since this density is given by $\rho = \psi \psi^* = \mathcal{R}^2$, satisfying

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \left(\rho \frac{\vec{\nabla} \varphi}{m} \right) = 0 \quad (6.19)$$

which is just (6.15) rewritten.

The surfaces of constant phase guide an infinite ensemble of particles, each with momentum $\vec{p} = m \vec{\nabla} \varphi$: *all* the trajectories defined by (6.18) are real in quantum mechanics. In all quantum systems, Many Worlds are present, though if we make a measurement, we will see only one particle. But we must keep in mind that in actuality, there are infinitely many particles — infinitely many histories — physically present. The same will be true in quantum cosmology.

But we will be aware of only one universe in quantum cosmology, so the requirement that classical mechanics hold exactly in the large in the present epoch can only mean that this single universe of which we are aware must obey exactly the classical Hamilton-Jacobi equation: that is, we must require that

$$\frac{\nabla^2 \mathcal{R}}{\mathcal{R}} = 0 \quad (6.20)$$

By requiring (6.20) to be imposed on the wave function of the universe in the present epoch, I have in effect unified the Many-Worlds Interpretation of quantum mechanics with Bohr's version of the Copenhagen Interpretation. In what is universally regarded as Bohr's definitive article on the Copenhagen interpretation — his paper “Discussion with Einstein on Epistemological Problems in Atomic Physics” in the P. A. Schilpp's *Albert Einstein: Philosopher-Scientist* [5] — Bohr never once claims that the wave function must be “reduced”; i.e., undergo non-unitary evolution, nor does he ever claim that macroscopic systems such as human beings are not subject to the unitary time evolution of atomic systems. Bohr instead asserts: “... it is decisive to recognize that, *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.*” ([5], p. 209, Bohr's italics) “... This recognition, however, in no way points to any limitation of the scope of the quantum-mechanical description ...” ([5], p. 211).

But quantum mechanics has unlimited validity only if it applies equally to human-size objects as well as to atoms, and thus the requirement that accounts of phenomena expressed at the human size level and larger must be in classical mechanical terms can only mean that the larger objects must obey classical and quantum laws simultaneously. And this is possible only if the human-size objects and larger obey

Schrödinger's equation and the classical H-J equation simultaneously, which requires that the boundary condition (6.20) hold.

But it is only required to hold at the present epoch (more precisely, after the first minute), and only on the largest possible scale, that of the universe as a whole. In other words, the Copenhagen Interpretation is to be regarded as something like the Second Law of Thermodynamics: it applies only to large scale systems, and it holds exactly only on the largest possible scale, the scale of the universe as a whole. The Many-Worlds Interpretation holds always, just as statistical mechanics holds always. But boundary conditions must be imposed on the MWI to yield exactly the Copenhagen Interpretation in the large, just boundary conditions must be imposed in statistical mechanics to yield exactly the Second Law in the Thermodynamic Limit.

However, as we shall see, imposing (6.20) today will yield a classical evolution from the initial singularity to the present day, thus justifying the use of classical field equations in the very early universe arbitrarily close to the initial singularity, as I have done in previous sections.

If \mathcal{R} is bounded above — as it would be if ψ were a function in a Hilbert space — equation (6.20) requires $\nabla^2 \mathcal{R} = 0$. This in turn implies (since \mathcal{R} is bounded above) that $\mathcal{R} = \text{constant}$. But the only allowed way in quantum mechanics to obtain $\mathcal{R} = \text{constant}$ is to extend the Hilbert space to a Rigged Hilbert space (Gel'fand triple) that includes delta functions. For example, when $V = 0$, a delta function in momentum space yields $\mathcal{R} = \text{constant}$, and the plane wave, which indeed satisfies the classical H-J equation, and indeed the trajectories which are everywhere normal to the constant phase surfaces are the straight lines with tangents proportional to the momentum vector.

It is important to emphasize that $\mathcal{R} = \text{constant}$ is going to yield a non-normalizable wave function, and that the only allowed non-normalizable wave function are indeed delta functions. For, as Böhm has pointed out [8], the most fundamental expression of the wave function, the Dirac kets, are themselves delta functions, so delta functions are physically real states that are actually physically more fundamental than the Hilbert space states. So we should not be surprised to find that the initial state of the universe is one of the most fundamental states.

The wave function of the universe $\Psi(R, \tau)$ in the mini-superspace described above is a function of two variables, the scale factor of the universe R and the conformal time τ .

If the initial boundary condition

$$\Psi(0, 0) = \delta(R) \tag{6.21}$$

$$\left[\frac{\partial \Psi(R, \tau)}{\partial R} \right]_{R=0} = 0 \tag{6.22}$$

is imposed, then the resulting wave function will have classical H-J trajectories for $\tau > 0$. (Boundary condition (6.22) is imposed in addition to the delta function condition (6.21) for the following reason. The wave function is assumed to have no support for $R < 0$. However, we cannot get this by imposing the DeWitt boundary condition $\Psi(0, \tau) = 0$, because it contradicts (6.21). But (6.22) is sufficient for self-adjointness of the SHO Hamiltonian on the half-line $R \in [0, +\infty)$; see [1] for a discussion.) The wave function satisfying boundary conditions (6.21) and (6.22) is just the Green's function $G(R, \tilde{R}, \tau)$ defined on the entire real line for the simple harmonic oscillator, with \tilde{R} set equal to zero. The wave function is thus

$$\Psi(R, \tau) = \left[\frac{6}{\pi L_P \sin \tau} \right]^{1/2} \exp \left[\frac{i6R^2 \cot \tau}{L_P^2} - \frac{i\pi}{4} \right] \tag{6.23}$$

where L_P is the Planck length. This wave function is defined only for a finite conformal time: $0 \leq \tau \leq \pi$. (The initial and final singularities *are* in the domain of the wave function!)

Notice that the magnitude of the wave function (6.23) is independent of the scale factor of the universe R . Since the scale factor plays the role of “spatial position” in the simple harmonic oscillator equation (6.12), we have $\nabla^2 R = 0$, and hence from the discussion on phase trajectories above, we see that the phase

trajectories for the wave function (6.23) are all the classical trajectories for a simple harmonic oscillator. That is, the phase trajectories are all of the form

$$R(\tau) = R_{max} \sin \tau \quad (6.24)$$

which are also all the classical solutions to the Einstein field equations for a radiation-dominated Friedmann universe.

We can also show that the phase trajectories are given by (6.24) by direct calculation. Since in the natural units $L_P = 1$, the phase φ is $\varphi = 6R^2 \cot \tau - \frac{\pi}{4}$, we have $\nabla \varphi = \partial \varphi / \partial R = 12R \cot \tau$. The tangents are defined by $p_R = \nabla \varphi$, which implies

$$\frac{1}{R} \frac{dR}{d\tau} = \cot \tau \quad (6.25)$$

using (6.6), $N = R$, and $\tau \rightarrow -\tau$. The solutions to (6.25) are (6.24).

With the boundary condition (6.21), *all* radii at maximum expansion, R_{max} , are present; all classical paths are present in this wave function. We thus see that, with the boundary condition (6.21), both the phase trajectories and the wave function begin with an initial singularity and end in a final singularity. In other words, with this wave function, the universe behaves quantum mechanically just as it does classically. The singularities are just as real in both cases. Conversely, we can run the calculation in reverse and conclude that in a SHO potential with $\mathcal{R} = \text{constant}$, we see that the universal wave function must have been initially a delta function.

c. Solution to Flatness Problem in Cosmology

Since $\rho(R(\tau)) = \psi\psi^* = \mathcal{R}^2$ measures the density of universes with radius R , for normalizable wave functions, it implies the Born Interpretation: the probability that we will find ourselves in a universe with size R is given by \mathcal{R}^2 . Similarly, if $\mathcal{R} = \text{constant}$, we are equally likely to find ourselves in a universe with any given radius. However, since $R > 0$, if we ask for the relative probability that we will find ourselves in a universe with radius larger than any given radius R_{given} or instead find ourselves in a universe with radius smaller than R_{given} , we see that the relative probability is one that we will find ourselves in a universe with radius larger than R_{given} , since $\int_{R_{given}}^{\infty} \mathcal{R}^2 d\mathcal{R} = +\infty$ while $\int_0^{R_{given}} \mathcal{R}^2 d\mathcal{R}$ is finite. Thus with probability one we should expect to find ourselves in a universe which if closed is nevertheless arbitrarily close to being flat. This resolves the Flatness Problem in cosmology, and we see that we live in a flat universe because (1) the Copenhagen Interpretation applies in the large, or equivalently, because (2) the quantum universe began as a delta function at the initial singularity, or equivalently, because (3) classical physics applies on macroscopic scales.

Notice a remarkable fact: although the above calculation was done using the Wheeler-DeWitt equation, the same result would have been obtained if I had done it in classical GR (in its Hamilton-Jacobi form), or even done it in Newtonian gravity (in its Hamilton-Jacobi form). Just as one can do FRW cosmology in Newtonian gravity, so one can also do FRW cosmology in quantum gravity. The conclusion is the same in all theories: the universe must be flat. This conclusion does not, in other words, depend on the value of the speed of light, or on the value of Planck's constant. In short, the flatness conclusion is robust!

d. Solution to the Standard Cosmological Problems: Homogeneity, Isotropy, and Horizon

One might think that quantum fluctuations would smear out the classical nature of spacetime near the initial singularity. Let me now prove that this is false; that in fact the consistency of quantum mechanics with general relativity requires these fluctuations to be suppressed. It is not the quantum fluctuations at the instant of their formation that gives rise to an inconsistency, but rather how such fluctuations would evolve in the far future. Fluctuations will in general yield mini-black holes, and it is the evolution of black holes, once formed, that give rise to inconsistencies.

Astrophysical black holes almost certainly exist, but Hawking has shown ([9]; [15], Section 7.3) that if black holes are allowed to exist for unlimited proper time, then they will completely evaporate, and unitarity will be violated. Thus unitarity requires that the universe must cease to exist after finite proper time, which implies that the universe has spatial topology S^3 . The Second Law of Thermodynamics says the amount of entropy in the universe cannot decrease, but it can be shown that the amount of entropy already in the CBR will eventually contradict the Bekenstein Bound [10] near the final singularity unless there are no event horizons, since in the presence of horizons the Bekenstein Bound implies the universal entropy $S \leq \text{constant} \times R^2$, where R is the radius of the universe, and general relativity requires $R \rightarrow 0$ at the final singularity. The absence of event horizons by definition means that the universe's future c-boundary is a single point, call it the *Omega Point*. MacCallum [11] has shown that an S^3 closed universe with a single point future c-boundary is of measure zero in initial data space. Barrow [12] has shown that the evolution of an S^3 closed universe into its final singularity is chaotic. Yorke [13] has shown that a chaotic physical system is likely to evolve into a measure zero state if and only if its control parameters are intelligently manipulated. Thus life (\equiv intelligent computers) almost certainly must be present *arbitrarily close* to the final singularity in order for the known laws of physics to be mutually consistent at all times. Misner [14] has shown in effect that event horizon elimination requires an infinite number of distinct manipulations, so an infinite amount of information must be processed between now and the final singularity. The amount of information stored at any time diverges to infinity as the Omega Point is approached, since $S \rightarrow +\infty$ there, implying divergence of the complexity of the system that must be understood to be controlled.

Let me now expand out the argument in the preceeding paragraph. First, let me show in more detail that unitarity (combined with the Hawking effect and the Bekenstein Bound) implies that the spatial topology of the universe must be S^3 . The argument I shall give is independent of the dynamics; it only depends on the basic structure of quantum mechanics and Riemannian geometry. A dynamical argument would be sticky if one does not want to make any a priori assumptions about the cosmological constant: a deterministic (globally hyperbolic) universe with a negative cosmological constant *always* will exist for only a finite proper time, whatever the spatial topology [17]. A dynamical proof for S^3 can be found in [18].

I have shown in Section 2 that the Bekenstein Bound, in the presence of particle horizons, implies that each region of space inside the particle horizon must have less than one bit of information when this spatial region becomes less than a Planck length in radius. Since less than one bit physically means that there is no information whatsoever in the region — that is, the laws of physics alone determine the structure of space — this region must be isotropic and homogeneous, because information must be given to specify non-FRW degrees of freedom. Now the Bekenstein Bound is not merely a local bound, but a global constraint, in the sense that it constrains a region with radius less than the Planck length to have zero information, rather than merely some open ball of with no a priori minimum size. But we can overlap these balls of Planck radius, to conclude that there is no information anywhere in the spatial universe at the Planck time.

Now a non-Compact FRW universe at the Planck time would still be of infinite volume, and thus would eventually create an infinite number of protons and neutrons, by the tunnelling process described in Section 5. Now Zel'dovich has shown that the lifetime of a proton to decay via the Hawking process is 10^{122} years (the actual value doesn't matter; it just has to be finite for my argument). If the universe held an infinite number of protons and neutrons, the probability is one — a virtual certainty — that at least one proton or neutron would decay via the Hawking process in the next instant after the Planck time, so the probability is one that unitarity would be violated. But unitarity cannot be violated, so the probability is one that the universe is spatially compact.

We can now apply the Bekenstein Bound to this compact universe, and note once again that the Bekenstein Bound is a global Bound; in particular, it implies that the amount of information is zero when the volume of the universe is the Planck volume. But if the universe were not simply connected, the topology itself would contain information, which is not allowed. Hence the universe must be spatially compact and simply connected. The homogeneity and isotropy of the universe, inferred above, implies that the spatial sectional curvatures are constant. Compactness implies ([16], p. 11) that spatially, the universe is complete. It is well-known (e.g., [16], p. 40) that the only complete, simply connected compact three-manifold with constant sectional curvature is the three-sphere.

e. Solution to Standard Model Hierarchy Problem

Since the validity of the Standard Model of Particle Physics — especially of the SM electroweak physics — is the essential assumption in this paper, I shall now further justify this assumption by pointing out that the standard quantum gravity combined with the Hawking black hole evaporation effect and the requirement of unitarity as discussed above, automatically resolves the Hierarchy Problem.

Recall that the Hierarchy Problem is explaining why the Higgs mass — and hence all the particle masses — are not dragged up to the Planck mass (or higher!) by the self-interactions as expressed by the renormalization group equation. Let us first note that the measurement of the top quark mass at around 175 GeV forces the SM Higgs boson mass to be around 200 GeV, because otherwise the SM Higgs potential would become unstable due to the higher order quantum corrections: the highest order term in the Higgs potential when the quantum corrections are taken into account is no longer $\lambda\phi^4$, but rather $C\phi^4 \ln(\phi^2/M^2)$ (to one loop order), and the constant C becomes negative, if the top quark mass and the Higgs mass becomes greater than about 175 GeV and 200 GeV respectively. (This renormalization group calculation assumes of course that some mechanism has already been found to prevent the one and higher loop self-energy corrections to the mass of the Higgs boson alone from dragging the Higgs mass to the Planck mass.)

The experimental fact that the SM Higgs vacuum potential is, given the observed top quark mass, only marginally stable is of fundamental significance: when combined with the Hawking effect, it provides the mechanism that solves the Hierarchy Problem.

Suppose on the contrary that the one and higher loop corrections to the mass of the Higgs boson increased the Higgs mass to greater than the allowed ~ 200 GeV. Then the mass of the Higgs would be pulled over the vacuum stability bound, and the mass of the Higgs would grow at least to the Planck mass, and the mass of the down quark would thus also increase to within an order of magnitude of the Planck mass. But this would mean that a neutron, with two valence down quarks, would become unstable via the Zel'dovich effect discussed above to the formation of a mini-black hole of near Planck mass, which would then evaporate via the Hawking process, violating unitarity. Hence, the one-loop and higher self-energy terms cannot act to increase the mass of the Higgs beyond the 200 GeV upper bound allowed by vacuum stability, since this would violate unitarity.

This also shows that the one and higher loop corrections, which are integrals over the energy in the loops, necessarily have a cut-off at an energy less than the Planck mass, a cut-off arising from quantum gravity. The cut-off is given by the requirement that the energy in the loop cannot increase to the level that would result in the formation of a mini-black hole even virtually. Thus in spite of naive appearance, this cut-off is Lorentz and gauge invariant. To see this, ignore for a moment all effects except for self-energy of a single particle. Then the upper bound to the value of the energy would be Planck energy, defined by the condition that no trapped surface of Planck energy is allowed to be formed, since such would give rise to a violation of unitarity. But the trapped surface condition is a Lorentz and gauge invariant condition.

Notice also that the upper bound to the energy in the loop integral actually depends on the proper time to the final singularity. If the upper bound were the Planck energy, then the loop correction would give rise to a Planck-size mini-black hole if the time before the final singularity were greater than the Planck time. Thus, in the earlier period of universal history — for example, now — the cut-off to the energy allowed in the loop integral must be less than the Planck energy. How much would be very difficult to calculate even given the knowledge of the length of time before the final singularity. But the cut-off must exist, and be less than the Planck energy at the present time.

An extension of this argument allows me to establish that the quantum gravity theory I have used in this section, namely the quantum gravity theory based on the Wheeler-DeWitt equation, is a completely reliable quantum gravity theory for the early universe, and in fact the early universe limit of more general quantum gravity theory that is both renormalizable and term by term finite. I refer to the quantum gravity theory that includes in the Lagrangian all terms consistent with $GL(2,R)$ symmetry of general relativity. It is well-known that if all terms consistent with this symmetry are included in the Lagrangian, then gravity is just as renormalizable as any other theory ([22], p. 506, pp. 517–519; [23], p.91–92; [24]). The problem has always been that there are an infinite number of such terms. This objection has been overcome by regarding

the resulting theory as an effective theory, with the higher order curvature terms coming in only at energies greater than the Planck energy, but with an apparent breakdown of the effective theory at energies greater than the Planck energy.

With the Hawking effect and unitarity, we see that no such breakdown occurs. Instead, the higher order curvature terms generate a more intense gravitational field than the first order Einstein Lagrangian, and thus would force a mini-black hole at a lower cut-off than the Einstein term. This means that in the current epoch of universal history, these higher order terms must be completely suppressed by unitarity. They will be important near the final singularity — when the time before the final singularity is less than the Planck time — but they are essentially suppressed at earlier times, in particular in the present epoch and near the initial singularity. So we can ignore these terms today and in the past, and thus the fact that adding an infinite number of terms to the Lagrangian necessarily involves an infinite number of constants that must be determined by experiment. The experiments need not be conducted, indeed cannot be conducted until the universe is within a Planck time of the final singularity. Measuring the values of these constants are among the infinity of measurements that must be carried out by life as the universe moves into the final singularity. At all times, however, the “effective” quantum gravity theory will be term by term finite, where I have placed the quotes because I claim that this standard quantum gravity theory can in fact be regarded as the true theory of quantum gravity.

Recognizing that the Hawking effect plus unitarity requires a Lorentz and gauge invariant upper bound to the energy in a loop integral — in other words, yields a Lorentz invariant ultraviolet cut-off — also solves the well-known problem of infinite particle production by time dependent gravitational fields. Expressing a quantized field as

$$\phi(\vec{x}, t) = (2\pi)^{-3/2} \int d^3 \vec{k} [A_{\vec{k}} \phi_{\vec{k}}(t) e^{i\vec{k} \cdot \vec{x}} + A_{\vec{k}}^\dagger \phi_{\vec{k}}^*(t) e^{-i\vec{k} \cdot \vec{x}}]$$

The operators $\phi_{\vec{k}}(t)$ and $\phi_{\vec{k}}^*(t)$ define what we mean by particles at time t . Given this definition, the corresponding definition at time t_0 is given by

$$\phi_{\vec{k}}(t_0) = \alpha_{\vec{k}}(t_0) \phi_{\vec{k}}(t) + \beta_{\vec{k}}(t_0) \phi_{\vec{k}}^*(t)$$

It was established by Parker more than thirty years ago that the necessary and sufficient condition for the particle number density to be finite is [19]

$$\int |\beta_{\vec{k}}(t_0)|^2 d^3 \vec{k} < \infty$$

Since in many cases of physical interest, $|\beta_{\vec{k}}(t_0)|^2$ drops off only as k^{-2} , this integral will diverge if the upper limit of the energy is infinity. However, the integral is a loop integral, and thus having the upper bound extend past the Planck energy would cause the spontaneous formation of a mini-black hole, which would immediately evapoate, violating unitarity. Once again, this ultraviolet cut-off does not violate Lorentz invariance, because what is giving the upper bound is the non-formation of a trapped surface, and whether a given 2-sphere is a trapped surface is a Lorentz invariant condition. So the definition of particle via the Hamiltonian diagonalization procedure (which is the definition used above) makes perfect sense given unitarity and the Hawking effect, so I must disagree with Fulling who opined in 1979 that no one should ever again take the idea of Hamiltonian diagonalization seriously ([19], p. 824).

It has recently been proposed ([26], [27]) that mini-black holes can be produced at the rate of one per second in the Large Hadron Collider, due to go on line in 2005. The above argument shows that this is absolutely impossible. No mini-black holes at all will be produced by the LHC, or by any other accelerator. My theory and indeed standard quantum gravity would be conclusively refuted by the unequivocal observation of mini-black holes in the LHC.

In the above I have made reference only to the down quarks in the Zel’dovich–Hawking effect. There is a reason for omitting the up quarks. Recall that the Zel’dovich upper bound is the average time required for two massive quarks to come within a Schwarzschild radius of each other, the Schwarzschild mass being assumed to be the Higgs quark mass. A particle with zero Yukawa coupling to the Higgs field would thus

have zero Schwarzschild radius, and thus two such particles would have an infinite time before coming within a Schwarzschild radius of each other. Thus any massless quark would not be subject to the Ze'dovich mechanism. I claim that the mass of the up quark is probably zero.

Recall that the other outstanding theoretical problem with the Standard Model of particle physics is the strong CP problem. Now that the B factories have seen CP violation, the solution of spontaneous CP violation is now ruled out, at least in the sense that all such models proposed to date predict that CP violation in B decay should be too small to be observed in the experiments where it was observed (I am grateful to Paul Frampton for a discussion on this point). The axion solution is generally considered to be ruled out by the required fine tuning in the early universe [20] — though I would rule it out because the axion has not been detected. The only remaining solution to the strong CP problem is for the mass of up quark to be zero.

Standard current algebra analysis (e.g. [23], p. 231) giving the ratios of quark masses in terms of the masses of various mesons indicate that the up quark has a non-zero mass, but Weinberg ([23], p. 458) points out that inclusion of terms second order in the strange quark mass might allow the up quark mass to vanish. Kaplan and Manohar for example claim [21] that $m_u = 0$ is allowed provided 30% of the squares of the meson masses arise from operators second order in the chiral symmetry breaking, and also that “The most striking feature of our result is that a massless up quark is not in contradiction with the past success of $SU(3) \times SU(3)$ chiral perturbation theory.” ([21], p. 2006).

Setting $m_u = 0$ solves the Strong CP Problem, and including standard quantum gravity effects in the Standard Model solves the Hierarchy Problem. Since these were the main theoretical problems with the Standard Model, we can be confident in the application of the Standard Model to the early universe — and also confident in the Standard Model’s claim that electromagnetism is not a fundamental field but instead is a composite of a $U(1)_R$ and an $SU(2)_L$ field.

In summary, the one and higher self-energy corrections to the Higgs boson indeed pull the Higgs boson mass up to a higher value — the loop integral pulls the Higgs (and top quark) mass up to the maximum value it can have consistent with vacuum stability. It cannot pull it up further than this, because a further value would violate unitarity via the Hawking effect. The Hawking effect, by imposing an upper bound to the energy (ultraviolet cut-off), an upper bound coming from the requirement that this stability bound be not exceeded, makes the Standard Model fully consistent.

f. Solution to the Cosmological Constant Problem

I have argued in [25] that the Hawking evaporation effect plus unitarity prevents the cosmological constant from being exceedingly large, and in fact requires that the effective cosmological constant, if ever it becomes small but positive, must eventually become zero or negative, since otherwise the universe even if closed would expand forever, resulting in the evaporation of the black holes which now exist, violating unitarity. What I shall now do is describe the physical mechanism that will eventually neutralize the observed currently positive effective cosmological constant.

It is well-known that the mutual consistency of the particle physics Standard Model and general relativity requires the existence of a very large positive cosmological constant. The reason is simple: the non-zero vacuum expectation value for the Higgs field yields a vacuum energy density of $\sim -1.0 \times 10^{26} \text{ gm/cm}^3 (m_H/246) \text{ GeV}$, where m_H is the Higgs boson mass. Since this is a negative vacuum energy, it is accompanied by a positive pressure of equal magnitude, and both the pressure and energy yield a negative cosmological constant. Since the closure density is $1.88 \times 10^{-29} \Omega_0 h^2 \text{ gm/cm}^3$, and observations indicate that $\Omega_0 = 1$ and $h = 0.66$, there must be a fundamental *positive* cosmological constant to cancel out the negative cosmological constant coming from the Higgs field. What we observe accelerating the universe today is the sum of the fundamental positive cosmological constant, and the negative Higgs field cosmological constant; this sum is the “effective” cosmological constant.

What we would expect is that these two cosmological constant would exactly cancel, and what must be explained is why they do not: the vacuum energy coming from the Higgs field — more generally, the sum of the vacuum energies of all the physical fields — is observed to be slightly less in magnitude than the

magnitude of the fundamental positive cosmological constant. What must be explained therefore, is why the vacuum energy sum is slightly less than expected.

I shall argue that the instanton tunnelling that has been shown in Section 5 to generate a net baryon number also results in the universe being in a false vacuum slightly above the true vacuum, where, as expected, the fundamental cosmological constant and the vacuum energy densities of all the physical fields do indeed cancel.

Recall that the instanton tunnelling works by non-perturbatively changing the global winding number of the $SU(2)_L$ field; the winding number is equal to the number of fermions in the universe. There is also a winding number associated with the $SU(3)$ color force, and the color vacuum — the θ -vacuum — is a weighed sum over all the winding numbers: $|\theta\rangle = \sum_n e^{-in\theta}|n\rangle$. The fact that θ is observed to be essentially zero is of course the “strong CP problem” which I resolved above.

There is no necessary connection between the winding numbers of $SU(2)_L$ and color $SU(3)_L$, but in fact $\pi_3(G) = \mathbb{Z}$ for any compact connected Lie group G , where $\pi_3(G)$ is the third homotopy group of G , expressing that there are non-trivial mapping of the three-sphere into G . There are thus *three* 3-spheres in cosmology and the Standard Model: (1) electroweak $SU(2)_L$ itself, (2) subgroups of color $SU(3)$ and (3) the spatial 3-sphere. I propose that the non-zero winding number due to mapping of $SU(2)_L$ into itself gives rise to a false vacuum in one or all of these three, and that the true vacuum corresponds to a winding number of zero.

This means that as long as the number of fermions minus anti-fermions remains constant on the spatial 3-sphere, the physical fields will remain in the false vacuum, the effective cosmological constant will remain positive, and the universe will continue to accelerate. Conversely, if instanton tunnelling occurs in reverse, so that the fermion number of the universe decreases, then the false vacuum will decrease to the true vacuum, a state which I have assumed has an energy density which cancels the positive fundamental cosmological constant. In the present epoch of universal history, the winding number remains constant — the tunneling probability is very small in the present epoch — and thus the sum of the false vacuum energy density and the fundamental cosmological constant, this sum being the *dark energy* since it is seen only gravitationally, is constant.

But in the long run, it cannot remain constant, since an unchanging positive dark energy would cause the universe to accelerate forever, violating unitarity when the observed black holes evaporate. Since the proton lifetime due to electroweak instanton tunnelling is greater than the solar mass black hole lifetime, something must act in the future to speed up the tunnelling probability.

I propose that life itself acts to annihilate protons and other fermions via induced instanton tunnelling. Barrow and I have established that the main source of energy for information processing in the far future will be the conversion of the mass of fermions into energy. Baryon number conservation prevents this process from being 100% efficient, but since the Standard Model allows baryon non-conservation via instanton tunnelling, I assume that some means can and will be found in the far future to allow life to speed up this process. So once again the existence of intelligent life in the far future is required for the consistency of the laws of physics, since in the absence of life acting to speed up fermion annihilation, the universe would accelerate forever, violating unitarity and incidentally extinguishing life.

Since a universe which expanded for a sufficiently long time would also extinguish life, a universe of the multiverse which has a radius of maximum expansion beyond a certain upper bound cannot develop structure: such structure would necessarily mean that the entropy is non-zero, and in Section 2, I showed, following Bekenstein, that in the presence of event or particle horizons, the entropy of the universe has to approach zero near a singularity, and only life can force the elimination of horizons. So for universes which have a radius at maximum expansion greater than this upper bound, both the initial and final singularities are Friedmann, with the entropy remaining zero for all of these very large universe’s history. Similarly, universes which have a radius sufficiently small can never develop structure, for life will never have time to evolve. (Universes whose radius at maximum expansion is less than the Planck length never develop structure, because the Bekenstein Bound never allows the generation of information at all; the entropy starts and remains zero from the Bekenstein Bound alone.)

So there is a narrow band of universes in the universe wherein entropy, structure, and life mutually exist. In these universes, the initial singularity is Friedmann, with zero entropy sufficiently close to the singularity, and with entropy that diverges as the final singularity is approached. This implies the Penrose condition on the initial and final singularity: an initial singularity is dominated by the Ricci curvature (Friedmann singularity) and a final singularity is dominated by Weyl curvature — dominate Weyl curvature is a necessary feature of Mixmaster oscillations which are required to eliminate event horizons.

The multiverse with the narrow band of universes containing entropy structure and life is pictured in Figure 6.1.

Figure 6.1: The Multiverse, and the entropy, structure forming and life band. For universes in the band, the initial singularity is isotropic and homogeneous with zero entropy, while the final singularity has infinite entropy, structure and Weyl curvature. Universes outside the band start and remain at zero entropy, never developing structure, Weyl curvature, or life.

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7. The SU(2) Gauge Field and the Higgs Field in the Present Day Epoch

a. Dynamics of the Pure SU(2) and Higgs Field

The Bekenstein Bound requires the Higgs field to initially have the value $\phi = 0$, as has been discussed in the previous section. Since the present day value is $\langle \phi \rangle = 246 \text{ GeV}$, the Higgs field must have rolled down the incline of its potential. It is usual to assume that this Higgs energy has long since been thermalized into the CMR, but as I have shown, the CMR gauge field component would have a Planck distribution in a FRW background whether it is thermal or non-thermal, I shall instead investigate the possibility that the Higgs energy has never been thermalized, but instead has been partly absorbed by the SU(2) gauge field, and partly decreased in density by the expansion of the universe.

Let us recall what the expansion of the universe must have been like so that the nucleosynthesis calculations are still valid, and so that the structure formation computer simulations are also still valid. During the nucleosynthesis era, the expansion rate must have been $R(t) \sim t^{1/2}$ corresponding to radiation domination — as discussed earlier, any massless gauge field will generate this behaviour if it is the dominant energy density in the universe. After decoupling, the expansion rate must change from the radiation domination rate to the matter dominated rate of $R(t) \sim t^{2/3}$ with a slight mixture of a Λ term, the change being required by structure formation simulations, which agree best with the Λ CDM model (actually, the best results are obtained by assuming that the CDM is slightly “warm”, but as we shall see, this will be allowed in my proposal for the dark matter). The nucleosynthesis data indicate that baryons are only a small fraction of the dark matter — which by definition is that “substance” which is responsible for the $R(t) \sim t^{2/3}$ expansion rate.

In my model there are only two fields outside of the baryons: the SU(2) gauge field and the Standard Model Higgs field. I shall now argue — but not prove — that it is *possible* for these two fields interacting together to produce the observation CMBR and the dark matter. (I have shown in Section 6 how the Standard Model vacuum naturally provides a non-zero cosmological constant; this I will take to be the dark energy).

Let us first consider the time evolution of the Higgs field by itself, and then consider its interaction with the SU(2) gauge field. For a general scalar field Lagrange density of the form $-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$, the equation of motion for the scalar field will be

$$\frac{d}{dt}\left(\frac{1}{2}\dot{\phi}^2 + V\right) = -3H\dot{\phi}^2$$

or

$$d\rho/dt = -3H(\rho + p) \tag{7.1}$$

where

$$\rho = \dot{\phi}^2 + V(\phi) \tag{7.2}$$

is the energy density of the scalar field, and

$$p = \dot{\phi}^2 - V(\phi) \tag{7.3}$$

is the pressure of the scalar field.

b. SOLUTION TO “DARK MATTER” PROBLEM:

WHAT IT IS AND HOW IT HAS ELUDED DETECTION

Turner has shown [3] that if we regard the scalar field as the sum of a rapidly oscillating part, and a slowly varying part, then a scalar potential of the form $V(\phi) = a\phi^2$, which is the approximate form of the SM Higgs potential in the present epoch, would give rise to an effective mass density that would drop off as R^{-3} , just as pressureless dust would. I conjecture that the combination of the SM Higgs field coupled to a pure $SU(2)_L$ field would naturally split into two fields that would appear to evolve independently, one dropping off as R^{-3} , and the other dropping off as R^{-4} . One would be the CMBR, and the other would be the dark matter. Recall that the Z boson has all the quantum numbers as a photon, and in fact can be made to form superpositions with photons. The interaction strength of the Z with fermions is stronger than the photon, and the only reason that the Z boson acts weakly is its large mass. Similarly, the candidate I am proposing as the dark matter will interact only weakly with fermions because it is basically a Z particle.

If this conjecture is correct, then the reason the dark matter has not been detected is because it must necessarily always be found in accompanied with $SU(2)_L$ pseudo-photons, and all the experiments to detect the dark matter have carefully been designed to eliminate all photon interactions.

Of course, the reason why such a possible dark matter candidate has heretofore not been considered is that it has been thought that the rapid oscillations of a SM Higgs field would quickly decay away ([3], section IV; [4]), into photons. I would conjecture that this is indeed what happens; the Higgs field decays into the $SU(2)_L$ field, which then passes the energy back into the Higgs field.

Let me emphasize (as if it needed emphasizing) that these are very counter-intuitive conjectures I am making, and I have given no mathematical evidence that the combined Higgs coupled to a pure $SU(2)_L$ field could in fact behave this way. I instead can only offer an *experimental* argument that something like this scenario must be in operation: it has been known for 35 years that ultra high energy cosmic rays propagate through the CMBR as if the CMBR were not present, and as I shall demonstrate in Section 9, this is possible if — and if the SM is true, only if — the CMBR has the properties of a pure $SU(2)_L$ field. And we have no laboratory experimental evidence that the SM is incorrect. The SM has passed every test we have put it through for the past 25 years.

c. WHY AN $SU(2)$ COMPONENT WOULD HAVE NO EFFECT ON EARLY UNIVERSE NUCLEOSYNTHESIS

The baryons, once created by the mechanism in section 5, would be in a Planck distribution gauge field, with thermal properties identical to the usual standard cosmological model. Recall that the interaction constants of the charged particles with the radiation field are relevant only to force the particles to also be in a thermal distribution like the radiation field. Thus, the reduced interaction strength of a pure $SU(2)_L$ field (discussed at length in Sections 8 and 9) would have no effect on the distribution of the particles and thus on nucleosynthesis. (The same would be true of the fluctuation spectrum observed in the acoustic peaks. As mentioned in Section 6, flatness requires a Harrison-Zel'dovich spectrum for the fluctuations, and the magnitude of the fluctuation spectrum is fixed by the requirement that the fluctuations be due entirely from the creation of baryons.

d. SUPPRESSING EARLY UNIVERSE PAIR CREATION, INVERSE AND DOUBLE COMPTON, and THERMAL BREMSSTRAHLUNG

I have argued in previous Sections that in the beginning, the universe must have contained nothing but a pure $SU(2)_L$ field. Even if this were true, one might think that this pure $SU(2)_L$ field would have long before the de-coupling time around a redshift of 1,000, this pure state would have thermalized into a normal

EM field. I cannot prove that there is *no* mechanism that would have resulted in the thermalization of the proposed pure $SU(2)_L$ field, but I can demonstrate that the standard ([5], [6], [7]) three main mechanisms of thermalization in early universe cosmology, namely pair creation, double compton scattering, and thermal bremsstrahlung actually will not thermalize a pure $SU(2)_L$ field.

An outline of the proof is simply to write down the Feynman Diagrams for all three processes, (actually only two; the Diagram for pair creation is essentially the same as the Diagram for Bremsstrahlung), and realize that each “pseudo-photon” of the pure $SU(2)_L$ field can couple *only* to left-handed electrons, and right-handed positrons. It is quickly noticed that the Diagrams violate conservation of angular momentum; all of these processes require a spin flip involving the same particle, and this is impossible. The no-go theorem is in all essentials the same as well-known decay asymmetry of the W boson.

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8. Detecting an $SU(2)$ Component In the Cosmic Microwave Background With the Original CMBR Detectors And Using a Penning Trap

a. Right-handed electrons Won't Couple to an $SU(2)$ CBR Component

Anyone contemplating a CBMR experiment should first familiarize him/herself with the basic experimental techniques. These are described in detail in Bruce Partridge's excellent book [3]. The experiments described in this section will be rather minor modifications of the basic CMBR experiments.

The main effect of the CBMR being a pure (or as we shall see, almost pure) $SU(2)_L$ gauge field is that in this case, **the CMBR will not couple to right-handed (positive helicity) electrons**, while standard electromagnetic radiation couples to electrons of both helicities with equal strength. All the experimental tests of the almost pure $SU(2)_L$ hypothesis which I shall propose in this section are based on this crucial property. But before reviewing the experimental tests, let me first discuss the question of coupling strength of the left-handed electrons with a CMBR which is pure $SU(2)_L$.

Recall that in the Standard Model, the $U(1)_R$ gauge field plays three roles. First and foremost, it allows the EM field to couple to right-handed electrons. Second, it forces a distinction between the Z^μ gauge field and the EM field 4-potential A^μ . Finally, it allows the unit normalizations of the $U(1)_R$ and the $SU(2)_L$ fundamental gauge fields B^μ and W_j^μ respectively to be carried over to the physical gauge fields Z^μ and A^μ . These latter two properties are usually termed the “orthogonality” and “normality” properties. The orthogonality and normality properties are at risk when there is no $U(1)_R$ gauge field at all, so I shall propose that the actual CMBR contains a small admixture of $U(1)_R$ to maintain these key properties. I would expect the energy density of the $U(1)_R$ component to be of the order of the energy density of the anisotropic perturbations in the CMBR, which would be the source of the small $U(1)_R$ component (recall that in the very early universe, the radiation field which is the sole matter constituent of the universe *must* be pure $SU(2)_L$).

In the Standard Model the gauge fields are related by

$$A^\mu = \frac{g_2 B^\mu + g_1 W_3^\mu}{\sqrt{g_1^2 + g_2^2}} \quad (8.1)$$

$$Z^\mu = \frac{-g_1 B^\mu + g_2 W_3^\mu}{\sqrt{g_1^2 + g_2^2}} \quad (8.2)$$

where g_1 and g_2 are respectively the $U(1)_R$ and the $SU(2)_L$ gauge coupling constants. It is clear from (8.1) and (8.2) that if the fundamental fields B^μ are normalized to unity, then so are A^μ and Z^μ , and also that the latter two fields are orthogonal if the former two are orthogonal. It is also clear that the real reason for the normalizations is to force the the EM field to couple with equal strength to both left and right handed electrons. But it is this equality that I am proposing does not exist in the case of the CMBR.

The coupling to electrons in the SM Lagrangian is

$$\bar{e}_L \gamma_\mu e_L \left[\frac{g_1}{2} B^\mu + \frac{g_2}{2} W_3^\mu \right] + \bar{e}_R \gamma_\mu e_R g_1 B^\mu \quad (8.3)$$

Suppose now that we set $B^\mu = 0$. Solving (8.1) for $W_3^\mu = (\sqrt{g_1^2 + g_2^2}/g_1)A^\mu$ — in which case a normalized W_3^μ does *not* yield a normalized A^μ and substituting this into (8.3) gives

$$e_{EM} \left(\frac{\alpha_2}{2\alpha} \right) A^\mu \bar{e}_L \gamma_\mu e_L \quad (8.4)$$

where $\alpha_2 = 1/32$ is the $SU(2)_L$ fine structure constant, and $\alpha = 1/137$ is the usual fine structure constant. So if we accept the normalization of (8.3), the coupling between electrons and the pure $SU(2)_L$ field would be increased relative to the usual e_{EM} . However, I would argue that either a small admixture of B^μ would force the usual coupling between the CMBR even if mainly $SU(2)_L$, or else the appropriate normalization to use in computing the interaction between a CMBR which is almost pure $SU(2)_L$ is to normalize A^μ even if it is mainly pure $SU(2)_L$. But I've gone through this calculation to point out that there *may* be a different coupling between a CMBR that is almost pure $SU(2)_L$, and the usual A^μ field CMBR. I doubt this possibility, because a stronger coupling would ruin the early universe nucleosynthesis results. The stronger coupling would also ruin the ultrahigh energy cosmic ray effect which I shall discuss in Section 9.

b. Detecting an $SU(2)$ Component Using Hans Dehmelt's Penning Trap

Hans Dehmelt's Penning Trap ([5], [6]) is the ideal instrument to test the idea that the CMBR will not interact with right-handed electrons. The basic structure of the Penning Trap is pictured in Figure 8.1.

Figure 8.1: the Penning Trap

(Figure 1 on page 17 of Dehmelt's Am. J. Phys. article).

Figure caption: Penning Trap (Taken from Dehmelt [6]) The electron orbit is a combination of vertical motion due to the electric field (pictured), and a circular cyclotron motion due to the magnetic field \vec{B}_0 . The pictured assembly is placed in an ultrahigh vacuum, and cooled to liquid helium temperature ~ 4 K. Cap to cap separation is about 0.8 cm.

In a Penning Trap, a single electron (or positron) is captured by a vertical magnetic field, and an electric field due to charges on a curved ring and two caps. In the Seattle Penning Trap, cap to cap separation is about 0.8 cm, the magnetic field \vec{B}_0 was 5 T. The magnetic field results in a circular cyclotron motion at $\nu_c = e\vec{B}_0/2\pi m_e = 141$ GHz, where e and m_e are the electron charge and electron mass respectively. The charge on the ring and the caps is adjusted to give a weak quadrupole field with potential well depth $D = 5$ eV, yielding an axial oscillation frequency of 64 MHz. (The electron feels a simple harmonic restoring force with spring constant $k = 2D/Z_0^2$), where $2Z_0$ is the cap to cap separation.

If two turns of nickel wire are wrapped around the ring electrode, the large applied magnetic field magnetizes this, and this "bottle field" interacts with the electron's magnetic moment, allowing the spin of the electron to be continuously measured. This "continuous Stern-Gerlach effect" forces the electron to be in one of its spin states, and it is possible to determine which one the electron is in, and to measure transitions between the two spin states.

The energy of the cyclotron motion of the electron is quantized, with energy

$$E_n = \left(n + \frac{1}{2}\right) h\nu_c$$

At 4 K, observations give $\langle n \rangle \approx 0.23$, and for intervals of about 5 second, the electron is observed in the $m = -1/2$ state or the $m = +1/2$ state ([5], p. 543). With $\nu_z = 64$ MHz, this means that if the state is chosen to be the $m = -1/2$, the electron will have positive helicity (be right-handed) for one-half the time for 128 million cycles — while the electron is moving down, and negative helicity (be left-handed) for the other half of the time; that is, when the electron is moving up.

The electron can undergo a spin flip $\Delta n = 0$, $m = +1/2 \rightarrow -1/2$. This is actually the result of two separate transitions: the transition $n = 0 \rightarrow 1$ is induced by the 4 K thermal radiation, and transition is

followed by the transition $(n = 1, m = -1/2) \rightarrow (n = 0, m = +1/2)$ induced by an applied rf field ([5], p. 543).

The key point to note that the thermal transition, were the electron with $m = -1/2$ to be immersed in the CMBR thermal field and were the CMBR to be a pure $SU(2)_L$ field, the thermal transition could occur only one-half of the time, that is, when it is moving up, when it has left-handed helicity. That is, **the “thermal” transition rate in a pure $SU(2)_L$ field would be one-half the transition rate in a pure electromagnetic heat bath.** Thus the Penning Trap can be used to determine whether the CMBR is indeed pure EM, or instead pure $SU(2)_L$, as I am claiming. An experiment to test this would introduce CMBR radiation via the gap between the ring electrode and the cap electrodes.

In the actual experiment, of course, the radiation from the cap and ring electrodes would in fact be thermal EM radiation, and this would induce transitions at all times in the electron’s motion. If there were no such radiation, the transition rate would increase from an average of 5 seconds to 10 seconds, but the actual transition rate would be proportional to the ratio of area between the electrodes to the area of the electrodes that face the cavity where the electron moves.

From the drawing of the Penning Trap in Figure 8.1, one might infer that this ratio would be quite small, but appearances can be deceiving. More precise drawings of the Penning Trap can be found in ([7], p. 235; [9], p. 108). I reproduce the more precise drawing from [7] below as Figure 8.2.

Figure 8.2: Scale Drawing of Penning Trap

In effect the electron is in the center of a spherical region whose center is the Penning Trap pictured in Figure 8.2. Let us approximate the area ratio as follows. Take a sphere of radius a , and intersect it with two coaxial cylinders, with the axis of both passing through the center of the sphere. Let the radii of the two cylinders be r_{in} and r_{out} , with $r_{in} < r_{out} < a$. Then the area of the sphere between the cylinders is

$$A = 4\pi a^2 \left[1 + \sqrt{1 - \left(\frac{r_{out}}{a}\right)^2} - \sqrt{1 - \left(\frac{r_{in}}{a}\right)^2} \right]$$

This area is a good approximation to the gap between the ring electrode and the cap electrode. If we feed the signal from the CMBR thorough only the gap between the upper cap and the ring electrode, then the available singla area would be 1/2 the above area. Making a rough measurement of the figure in ([9], p. 108), I obtained $A/4\pi a^2 = 0.49$, and if this is accurate, as much as 1/4 of the “thermal” radiation inducing a state transition can be a signal from outside the Penning Trap, assuming only the upper gap is used (as the terminus of a circular wave guide).

In other words, the outside signal will in effect be transmitted through a coaxial wave guide, for which the gap between the upper cap and the ring electrode is the terminus. Recall that a coaxial wave guide can transmit TE and TM, as well as TEM waves. The power flow through a coaxial wave guide is calculated by all physics graduate students([1], p. 385) to be

$$P = \left[\frac{c}{4\pi} \right] \sqrt{\frac{\mu}{\epsilon}} \pi r_{in}^2 |H_{in}|^2 \ln \left(\frac{r_{out}}{r_{in}} \right) = \left[\frac{c}{4\pi} \right] \sqrt{\frac{\mu}{\epsilon}} \pi r_{out}^2 |H_{out}|^2 \ln \left(\frac{r_{out}}{r_{in}} \right)$$

Or, if $|H| = |E|$, as it will be for TEM waves, and we assume Gaussian units (in which case the factor in brackets comes into play and $\mu = \epsilon = 1$ for a vacuum or air wave guide), the power passing through the wave guide will be

$$P = c\rho_{in}\pi r_{in}^2 \ln \left(\frac{r_{in}}{r_{out}} \right) \approx 2c\rho_{av}A$$

where ρ_{in} , ρ_{av} , and A are the energy density at the inner radius, the average energy density in the annulus, and the area of the open annulus respectively, and I have assumed that $(r_{out} - r_{in})/r_{in} \ll 1$. So the power flow from the out side is just the flow on would expect through an opening of the size of the gap; the walls have no significant effect.

Of course, the signal from outside the Penning Trap will consist of radiation from several sources, only one of which will be CMBR. The other radiation sources will be EM field radiation, and will couple to right-handed electrons. The various other sources are discussed at length in ([3], pp. 103–139), and a shorter introduction to these other sources can be found in the original papers, e.g. ([10], [11]). Remarkably, the main non-CMBR is 300 K radiation from the ground, and if the detector is shielded from this radiation — easy to do with metal reflectors preventing the ground radiation from reaching the detector antenna — then the other radiation sources all have a radiation temperature of the order of a few degrees K, and there are methods to measure them independently, and thus subtract them out.

For example, the atmosphere has a zenith temperature of about 4 K, but as the optical depth depends on the angle z from the zenith as $\sec z$, the atmosphere temperature goes as $T_{atm}(z) = T_{atm}(0) \sec z$, and thus by making a series of measurements of the total energy received by the antenna at several angles z , the energy of the atmosphere can be subtracted out (details [3], pp 120–121).

Since the transition probability ($n = 0 \rightarrow n = 1$) depends on the square of the cyclotron frequency [7], the transition rate due to the CMBR will be too low unless the frequency looked at is near the 5 T cyclotron frequency $\nu_c = 141$ GHz. This is much higher than the window of 3 to 10 GHz used in the classical CMBR measurements. However, there is an atmospheric window at 0.33 cm, or 91 GHz, sufficiently near the 5 T cyclotron frequency that the transition rate would be reduced only by a factor of $(91/141)^2 = 0.42$, and the required 3.2 T Penning Trap magnetic field should be easy to achieve. The CMBR observation at 91 GHz, however, is best conducted at high altitudes (the first CMBR measurement was conducted at the High Altitude Observatory at Climax Colorado which was at an altitude of 11,300, ft. The instrument was actually tested at Princeton University, where it was built, but even in the winter, the water vapor at Princeton made the measurement of the $\sec z$ atmosphere contribution difficult to eliminate (D.T. Wilkinson, private communication). But in principle, the 91 GHz CMBR measurement could be done (though with difficulty) even at Seattle or Cambridge, MA, where Penning Traps are in operation. It would better done with the operational Penning Trap at Boulder CO, which is already at a high altitude, and the water vapor is naturally low.

Although I have emphasized that the first effect one should search for with the Penning Trap is the reduction in transition rate due to the fact that the CMBR can interact with the Penning Trap electron only for 1/2 the time, an even better test would be to observe that the transition occurs only in that part of the electron's orbit when the electron is left-handed, for example when a spin down electron is moving up, and when a spin up electron is moving down. With positrons, the situation would be reversed: since the $SU(2)_L$ field can couple only to right-handed positrons, a spin up positron should be able to interact with a pure $SU(2)_L$ CMBR only when the positron is moving up, and a spin down positron would be able to interact only when it was moving down. However, such a measurement would be difficult given the standard voltage between the cap and ring electrodes, which yield the 64 MHz vertical motion frequency.

c. Detecting an $SU(2)$ Component With the Original CMBR Detectors with Filters

As I said above, the Penning Trap is the ideal instrument to determine whether or not the CMBR is indeed a pure $SU(2)_L$ field, or simply an EM field. Unfortunately, setting up a Penning Trap to look for the expected difference is quite an expensive proposition; a series of e-mails between myself and Hans Dehmelt's group indicated that it would take \$ 250,000 and more to set up such an experiment, to say nothing of the difficulty of moving the instrument to the best location, a dry desert high altitude plateau. For this reason, it would be nice if a quick and inexpensive test of the pure $SU(2)_L$ hypothesis could be found. In this subsection, I shall outline such an experiment, but I should caution the reader that the proposed apparatus depends on estimates on the behaviour of electrons in conductors and semi-conductors when an $SU(2)_L$ field interacts with electrons in such a material, and these estimates might not be reliable. So a null result *might* not rule out the pure $SU(2)_L$ hypothesis. On the plus side, a positive result *would* confirm the $SU(2)_L$ hypothesis, and the quick and dirty experiment I shall now propose can be done with a simple modification of the original apparatus set up by the Princeton group in 1965 to detect the CMBR. Even if the original

apparatus no longer exists, it can be re-built for the cost of at most a few thousand dollars, and a single day's observation should suffice to see if the effect is present.

The basic physical effect I shall use is as follows. Since a pure $SU(2)_L$ CMBR field will not couple to electrons with positive helicity, a CMBR wave will penetrate deeper into a conductor than an ordinary EM wave, since in a conductor at room temperature the conduction electrons have on the average zero net helicity: half on the average have positive helicity and the other half have negative helicity. I shall now show how to use this fact to confirm that the CMBR is a pure $SU(2)_L$ field.

The transmission coefficient for EM waves into an infinite conducting plate with vacuum on either side has been calculated by Stratton ([15], pp. 511–516). I shall repeat his derivation because it will allow me to point out some of the problems that may arise using the filter experiment rather than the Penning Trap to detect a pure $SU(2)_L$ CMBR.

Let us, following Stratton, imagine that we have three arbitrary homogeneous media labeled (1), (2), and (3), with dielectric constants $\epsilon_1, \epsilon_2, \epsilon_3$, magnetic permeabilities μ_1, μ_1, μ_1 , and propagation factors k_1, k_2 , and k_3 respectively. The thickness of the intermediate medium (2) will be d . In medium (1), we only have magnitudes of the incident and reflected waves:

$$E_i = E_0 e^{ik_1 x - i\omega t}, \quad H_i = \frac{k_1}{\omega \mu_1} E_i$$

$$E_r = E_1 e^{-ik_1 x - i\omega t}, \quad H_r = -\frac{k_1}{\omega \mu_1} E_r$$

The EM field in the middle medium (2) will contain wave which are moving to the right and waves which are moving to the left:

$$E_m = (E_2^+ e^{ik_2 x} + E_2^- e^{-ik_2 x}) e^{-i\omega t}$$

$$H_m = \frac{k_2}{\omega \mu_2} (E_2^+ e^{ik_2 x} - E_2^- e^{-ik_2 x}) e^{-i\omega t}$$

and finally the wave transmitted into medium (3) is

$$E_t = E_3 e^{ik_3 x - i\omega t}, \quad H_t = \frac{k_3}{\omega \mu_1} E_t$$

From these equations we see one possible problem in using a filter rather than a single electron to interact with an $SU(2)_L$ field: there may be numerous reflections at the boundaries between the three media, and these many reflections may cause a pure $SU(2)_L$ field to be converted into an EM field, through interacts between left and right handed electrons in the media themselves.

I shall assume that this does not occur. Stratton points out that the boundary equations are easier manipulate in terms of the following quantities:

$$E_j = \pm Z_j H_j, \quad Z_j \equiv \frac{\omega \mu_j}{k_j}, \quad Z_{jk} \equiv \frac{Z_j}{Z_k} = \frac{\mu_j k_k}{\mu_k k_j}$$

The boundary conditions yield four equations between five amplitudes:

$$E_0 + E_1 = E_2^+ + E_2^-$$

$$E_0 - E_1 = Z_{12}(E_2^+ - E_2^-)$$

$$E_2^+ e^{ik_2 d} + E_2^- e^{-ik_2 d} = E_3 e^{ik_3 d}$$

$$E_2^+ e^{ik_2 d} - E_2^- e^{-ik_2 d} = Z_{23} E_3 e^{ik_3 d}$$

The transmission coefficient is $T = |E_3/E_0|^2$, so it is only necessary to solve for

$$T = \frac{E_3}{E_0} = \frac{4e^{-ik_3 d}}{(1 - Z_{12})(1 - Z_{23})e^{ik_2 d} + (1 + Z_{12})(1 + Z_{23})e^{-k_2 d}}$$

I shall simply by setting $\mu_1 = \mu_2 = \mu_3 = \mu_0$, where μ_0 is the magnetic permeability of free space, and assume that $\epsilon_1 = \epsilon_3 = \epsilon_0$, where ϵ_0 is the dielectric constant of free space. We have in this case

$$k_1 = k_3 = \frac{\omega}{c}$$

where c is the speed of light in a vacuum, and

$$k_2 = \alpha + i\beta$$

where

$$\alpha = \frac{\omega}{c} \left[\left(\frac{\mu_2}{\mu_0} \right) \left(\frac{\epsilon_2}{\epsilon_0} \right) \sqrt{1 + \left(\frac{\sigma}{\epsilon_2 \omega} \right)^2} + 1 \right]^{1/2}$$

and

$$\beta = \frac{\omega}{c} \left[\left(\frac{\mu_2}{\mu_0} \right) \left(\frac{\epsilon_2}{\epsilon_0} \right) \sqrt{1 + \left(\frac{\sigma}{\epsilon_2 \omega} \right)^2} - 1 \right]^{1/2}$$

where σ is the conductivity of medium (2). (The formulae for α and β simplify in the cases of good conductors and bad conductors — see ([1], pp. 297 — but with Mathematica, it's just as easy to use the general formulae).

the electrical conductivity is given by

$$\sigma = \frac{ne^2\tau}{m_e}$$

where n is the number of conduction electrons per unit volume, e is the electron charge, τ is the relaxation time, and m_e is the electron mass. This formula for the conductivity will be valid unless the EM frequency is greater than 5×10^4 GHz. The key fact I shall use is that as described above,

$$n_{SU(2)_L} = \frac{1}{2} n_{EM}$$

since an $SU(2)_L$ field would interact with only half of the conduction electrons.

For conductors and semi-conductors, almost all of an incident EM (and $SU(2)_L$) wave will be reflected unless the thickness d of the filter (medium (2)) is small relative to the skin depth.

The range of wavelengths for viewing the CMBR at sea level is 3 cm to 10 cm, or 3 GHz to 10 GHz; the upper frequency is determined by absorption from water vapor, and the lower frequency by background sources in the Galaxy. The skin depth of Copper is 1.2 microns at 3 GHz, $\mu_{Cu}/\mu_0 = \epsilon_{Cu}/\epsilon_0 = 1$, and $\sigma = 5.80 \times 10^7$ mho/meter, for which the transmission coefficient is only $T = 0.0013$ even if $d = 10^{-3}$ (skin depth), or 12 angstroms — the size of the copper atom. Clearly, no good conductor with a conductivity comparable in magnitude to copper would yield a detectable signal. We want to find a material with a transmission coefficient greater than a few per cent for a thickness of at least 100 atoms, so that we can trust the continuum approximation for medium (2).

Graphite, with $\mu_{Cu}/\mu_0 = \epsilon_{Cu}/\epsilon_0 = 1$, and $\sigma = 1.0 \times 10^5$ mho/meter, is marginal. With a thickness of $d = 10^{-3}$ (skin depth), or 290 angstroms — near the 100 atom thickness — the transmission coefficient is $T = 0.23$, while for $d = (1/200)$ (skin depth), or 1450 angstroms, the transmission coefficient is $T = 0.024$, which may be detectable. The idea would be to allow the CMBR to pass through the filter, and with this latter thickness, a graphite filter would transmit 2.4% of the CMBR flux. The test would be to measure the CMBR and a reference 2.726 K reference EM source with the filter, using the above formulae for the relative conductivities and for α and β . More flux will be detected from the CMBR if the pure $SU(2)_L$ hypothesis is correct.

I shall illustrate this difference with the material I think has the best chance of giving an acceptable filter, namely the element Germanium, for which $\sigma = 2.1$ mho/meter — this conductivity is the same at 9.2 GHz as at zero frequency — and $\mu_{Ge}/\mu_0 = 1$. Unfortunately, we have $\epsilon_{Ge}/\epsilon_0 = 16.2$ at 300 K (the ratio is 16.0 at 4.2 K), and this non-vacuum polarizability needs special consideration. According to the simple models of molecular polarizability given in ([1]. pp. 152–158), the small temperature difference in ϵ_{GE} indicates that most of the molecular polarizability is due to a distortion in the charge distribution by the applied electric field generating an induced dipole moment in each molecule. Since the electric field will act on *bound* electrons rather than *free* conduction electrons, the distortion on the left handed electrons will be almost instantaneously transmitted to the right handed electrons, with the result that the molecular polarizability would be the same for both an EM field and for a pure $SU(2)_L$ field, and thus the dielectric constants would be the same in both cases; on the conductivities would be different, and the ratio of the two will be 1/2, as described above. (Even if the effective dielectric constant were to be different for an EM and a pure $SU(2)_L$ field, the discussion on page 154 of [1] makes it clear that it would vary in a more complicated way than the conductivities, and thus the transmission coefficients would be measurably different for the two types of gauge field. For example, equation 4.70 of [1] yields $N\gamma_{mol} = 1/5$ for Germanium in an EM field; if $N\gamma_{mol} = 1/10$ in an $SU(2)_L$ field, equation 4.70 yields $\epsilon/\epsilon_{\text{epsilon}_0} = 3.16$, a substantial change from 16.2.)

For a thickness of 1.6 millimeters, we have

$$T_{EM}(3 \text{ GHz}) = 0.106$$

$$T_{SU(2)_L}(3 \text{ GHz}) = 0.132$$

which gives

$$\frac{T_{SU(2)_L}}{T_{EM}}(3 \text{ GHz}) = 1.24$$

or a 24% greater flux if the CMBR is a pure $SU(2)_L$ field.

The corresponding transmission coefficients at 10 GHz is

$$T_{EM}(10 \text{ GHz}) = 0.176$$

$$T_{SU(2)_L}(10 \text{ GHz}) = 0.296$$

which gives

$$\frac{T_{SU(2)_L}}{T_{EM}}(10 \text{ GHz}) = 1.68$$

or a 68% greater flux if the CMBR is a pure $SU(2)_L$ field. (The fluxes are greater at a higher frequency than at the lower frequency — opposite to what we would expect for a good conductor — because Germanium is a semi-conductor.)

A typical CMBR radiometer is pictured in Figure 8.3

Figure 8.3: Figure 4.14 of Partridge, p. 126

The central sky horn could be covered with the filter — mounting the filter inside the radiometer would probably work, but there would be the risk of $U(1)_R$ field generation by second order effects in the wave guide. But Germanium is expensive, so I would risk setting the filter inside a coaxial wave guide.

It is important to note that the above calculations explain why the effect I am predicting have never been seen before. If the parts of the radiometers designed to absorb CMBR are thicker than the skin depth for the radiation absorbers — and indeed all such absorbers are much thicker — then all the CMBR would be absorbed even though the effective conduction electron density is only 1/2 of the conduction electron density seen by the usual EM field. In fact, the CMBR absorbers are made much thicker than the skin depth precisely to insure that all incident radiation will be absorbed, and this thickness hides the effect I am predicting. In 1938, the German physicist G. von Droste bombarded uranium with neutrons, but carefully covered the uranium with metal foil in order to eliminate alpha particles which were expected to occur. It worked; but the foil also eliminated fission fragments, which have a shorter range in metal foils than alpha particles. In the opinion of historians ([4], p. 41) the metal foils cost von Droste the Nobel Prize for the discovery of nuclear fission. The same experimental technique also cost ([8], pp. 7–8) Enrico Fermi a Nobel for the discovery of nuclear fission. Fermi began the bombardment of uranium with neutrons in 1935, but like Droste he covered his samples with aluminum foil. Once again the foil absorbed the fission fragments that in the absence of the foil, Fermi’s instruments would have clearly seen. In the end, fission was discovered in 1939 by Otto Hahn and Lise Meitner, who used not standard particle detectors, but instead the methods of radiochemistry. All investigations of the CMBR to date have used too thick a “foil” and thus have missed the important effect I am predicting. Actually, as we shall be in Section 9, there are measurements of the CMBR that are analogous to radiochemistry in that the instruments used do not “cover up” the effect: these “instruments” are ultrahigh energy cosmic rays, and I shall argue that they have already detected the effect I am predicting.

Actually, it is possible that some early experiments detected the expected difference between an EM CMBR and an $SU(2)_L$ CMBR. Two groups, one headed by Gish and the other by Woody and Richards, measured the CMBR using filters analogous to the filters I have discussed above, and they detected ([3], p. 142) an excess flux above what was expected for a 2.7 K blackbody EM field. The Woody and Richards experiment also saw a lower flux than a 2.7 at lower frequencies (below the 2.7 blackbody peak), which is just what we would expect from an $SU(2)_L$ CMBR field, as I discussed above.

d. Other Means of Detecting an $SU(2)$ CMBR Component

The Penning Trap is not the only way of observing an interaction between the CMBR and a single electron. A Rydberg atom — which are atoms in states with a high principal quantum number n — can undergo transitions induced by blackbody radiation at liquid helium temperatures ([12], chapter 5), and hence such atoms could be used to detect the effect I am predicting, provided the Rydberg atom can fix its transition electron in a definite spin state when the atom’s motion is observed.

It has been occasionally suggested (e.g., [14]) that the devices which allowed the observation of Bose-Einstein condensation — the magneto-optical trap (MOT) — might be able to observe the CMBR. A MOT is designed to excite hyperfine transitions, and the collective motion of the atoms in the trap is observable, so in principle the effect I am predicting *might* be observable with a MOT. The problem with using a MOT is that the cross-section for the hyperfine transitions is so low that MOTs usually are set up at room temperature, and at 300 K, the tail of the blackbody radiation in the 3 to 10 GHz range is some two orders of magnitude greater than the 2.7 K distribution. The fact that low temperature experiments can be carried out in MOTs at room temperature is the reason MOTs are so useful. But this very advantage of MOTs for many experiments makes the MOT useless for observing the CMBR. The opinion of Cornell and Wieman ([13], p. 49) is that “... it is difficult to imagine that [blackbody radiation] will ever be important for trapped atoms.”

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9. Has an SU(2) CBR Component Already Been Detected? Ultrahigh Energy Cosmic Rays

a. Why the Ultrahigh Energy Cosmic Rays Should Not Exist, But Yet They Do Exist

In regard to ultra high energy (UHE) cosmic rays — particles above 10^{19} eV — Alan Watson of the University of Leeds recently described the observations succinctly: “They ... are extremely hard to understand: put simply — they should not be there” [2]. The reason UHE cosmic rays should not be there is that they are too energetic to be confined to the galaxy by the galactic magnetic field, yet they cannot propagate very far in intergalactic space because their energy would be absorbed by collisions with CMBR photons.

The detailed mechanism preventing the existence of UHE cosmic rays was discovered by Kenneth Greisen in 1966, shortly after the discovery of the CMBR. Greisen pointed out that protons of sufficiently high energy would interact with the CMBR, producing pion, resulting in a cut-off to the UHE cosmic ray spectrum. Even in his paper of 35 years ago, he pointed out that “... even the one event recorded at 10^{20} eV appears surprising. ... [the CMBR] makes the observed flattening of the primary spectrum in the range $10^{18} - 10^{20}$ eV quite remarkable.” ([9], p. 750). Since Greisen wrote his paper, the experiments have become even more inconsistent with the existence of the CMBR, as illustrated in Figure 9.1:

Figure 9.1

Figure taken from Figure 7.1, page 119 of [1].

Figure 9.1 Caption: UHE Energy Spectrum observed over the past 7 years with the the AGASA detector in Japan. The dashed curve is the expected rate with a uniform cosmological distribution, but with the expected interaction of protons and the CMBR. (taken from [1], p. 119 and [2], p. 819; figure due to M. Takeda of the Institute for Cosmic Ray Research, University of Tokyo [12].) The upper group of 3 events is 6σ above the theoretical curve.

The AGASA array in Japan has detected 461 events with energies above 10^{19} eV, and 7 events above 10^{20} eV. The Haverah Park array in England has detected 4 events above 10^{20} eV, the Yakutsk array in Siberia has detected 1 event above 10^{20} eV. The Volcano Ranch array in New Mexico has detected 1 event above 10^{20} eV ([1], p.118). So four different groups over the past 35 years have repeatedly seen these particles that shouldn't be there. The consensus of the experimental cosmic ray physicists is that the Greisen cut-off does not exist ([2], p. 818).

At energies above 10^{20} eV, there is no clustering in arrival directions ([1], p.121; [2], p. 819). This is illustrated in Figure 9.3, which gives the arrival directions of 114 events at energies above 4×10^{19} . At such energies, the gyromagnetic radius is comparable to the size of the Galaxy, so UHE cosmic rays should be extragalactic. The only obvious source within 30 Mpc (see below) is the Virgo Cluster, but there is no clustering in this direction. (Intergalactic magnetic fields are too low to effect the arrival direction within 30 Mpc.)

Figure 9.2

Figure taken from Figure 2, page 820 of [2].

This blatant inconsistency between the observed energies of some UHE cosmic rays and the global existence of the CMBR has lead a number of physicists to propose modifications in the known physical laws. However most physicists, myself included, “... believe that, within certain well-defined physical limitatons, the laws of physics as we know then can be trusted,” to quote the words of Malcolm Longair, the Astronomer Royal of Scotland ([6], p. 333). What I shall now show is that there is a mechanism, using only the firmly tested physical laws, whereby UHE protons can propagate cosmological distances through the CMBR — provided the CMBR is not the complete EM field, but rather only the $SU(2)_L$ part of the EM field.

b. How an $SU(2)$ Componet to the CBR Would Permit UHE Cosmic Rays to Propagate

Recall that CMBR blocks the propagation of UHE cosmic rays via the GZK effect [9]: protons comprising the UHE particles collide with a CMBR photon, resulting in pion production. The reactions are

$$\gamma + p \rightarrow \Delta^+ \rightarrow \pi^0 + p \quad (9.1)$$

$$\gamma + p \rightarrow \Delta^+ \rightarrow \pi^+ + n \quad (9.2)$$

$$\gamma + p \rightarrow \Delta^{++} + \pi^- \rightarrow \pi^- + \pi^+ + p \quad (9.3)$$

where p , n , and π are the proton, neutron, and pion respectively. The reaction cross-sections are dominated by Δ particle resonances ([5], [9]). Of the total cross-section for (9.2) of 300 microbarns at peak $E_\gamma = 320$ MeV, 270 comes from the Δ resonance ([5], p. 14). Of the total cross-section for (9.3) of 70 microbarns at peak $E_\gamma = 640$ MeV, virtually all comes from the Δ^{++} resonance ([5], p. 14). Of the total cross-section for (9.1) of 250 microbarns at peak $E_\gamma = 320$ MeV, 140 comes from the Δ resonance ([5], p. 13). For (9.3), virtually all the total cross-section for photon energies less than the peak also comes from the Δ^{++} resonance. For the other reactions, the rest of the total cross-section arises from a photoelectric term ([5], p. 14).

However, if the CMBR consists not of electromagnetic radiation, but is instead a pure $SU(2)_L$ field, then the Δ resonance cannot occur. The reason is simple: the Δ particle originates from a proton by a quark spin flip ([5], p. 16), but since a $SU(2)_L$ field couples only to a left-handed quark, it cannot induce a quark spin flip: a pure $SU(2)_L$ photon would not couple to a right-handed quark at all, and a left-handed quark would have the handedness unchanged by the interaction.

Furthermore, the photoelectric term would be reduced because only a fraction of the electric charge on the quarks would interact with a pure $SU(2)_L$ field. If for example, a proton were to have only its down valence quark left handed, then its effective electric charge would be $-1/3$ rather than $+1$. Since the photo cross-sections are proportional to the (charge)⁴ (the square of the classical electron radius, with the “electron” having a charge of $-1/3$), the photo cross-section would be reduced by a factor of $1/81$ from its value for an electromagnetic CMBR. Even if one of the up quarks were left-handed, the photo cross-section would be reduced by a factor of $16/81 \approx 1/5$.

The net effect on the total-cross-sections (for the down valence quark case) is to reduce the cross-section for pion production from $SU(2)_L$ photons $\sigma_{SU(2)}$ from its value σ_{EM} that we would have if the CMBR were to be a normal electromagnetic field:

$$\sigma_{SU(2)}^{p\pi^0} = \frac{1}{150} \sigma_{EM}^{p\pi^0} \quad (9.4)$$

$$\sigma_{SU(2)}^{n\pi^+} = \frac{1}{810} \sigma_{EM}^{n\pi^+} \quad (9.5)$$

$$\sigma_{SU(2)}^{p\pi^+\pi^-} = 0 \quad (9.6)$$

The mean free path L_{MFP} for an UHE proton is

$$L_{MFP} = (\sigma^{p\pi^0} N_{photon})^{-1}$$

Using $N_{photon} = 5 \times 10^8 \text{ m}^{-3}$ we would get $L_{MFP} \approx 10^{23} \text{ m} \approx 3 \text{ Mpc}$ ([6], p. 340) if we used $\sigma_{EM}^{p\pi^0}$. Using (9.4), however, we get

$$L_{MFP} = 450 \text{ Mpc} \quad (9.7)$$

which means that UHE protons can propagate through the intergalactic medium as if the CMBR were not there. This becomes even more obvious when we consider that the fractional energy loss due to pion creation is $\Delta E/E \approx m_\pi/m_p \approx \frac{1}{10}$, so the propagation distance would be more than 4 Gpc, which is truly a cosmological distance.

If pion production is no longer significant, then one could wonder about the removal of UHE proton via electron-positron pair production. As is well-known ([6], p. 341), the cross-section for pair production from the collision of a UHE proton with an EM CMBR photon is actually greater than the cross-section for pion production, but the much smaller mass of the pair means that with EM CMBR photons, the energy loss per collision is less by a factor of $(m_\pi/m_e)(\sigma^{p\pi}/\sigma_{pair}) \approx 6$. I have shown in an earlier section of this paper that pair production is not possible via collision of two $SU(2)_L$ photons, but it is not necessary to investigate whether this would be the case for the collision of such a CMBR photon with a proton. For the cross-section for pair production is proportional to $\alpha_{EM} r_e^2$, and thus the cross-section for pair production would be also reduced by at least a factor of $\frac{1}{150}$ by the effective charge reduction mechanism that reduced the pion production cross-section.

The energetics of the UHE cosmic rays are completely consistent with a cosmological source ([6], pp. 341–343). The local energy density of cosmic rays with energies above 10^{19} eV is 1 eV m^{-3} . Following [6], p. 342), let us assume that each source of UHE cosmic rays generates a total cosmic ray energy of E_{CR} over the age of the universe, and that N is the spatial density of these sources. In the absence of losses, the energy density of UHE cosmic rays would be

$$\rho_{CR} = E_{CR} N$$

For strong radio galaxies, $N \approx 10^{-5} \text{ Mpc}^{-3}$, so we would have to have $E_{CR} \approx 5 \times 10^{53} \text{ J}$, or about $3 \times 10^7 M_\odot$ of energy in the form of UHE cosmic rays produced per source over the history of the universe. Given that the black hole cores of many radio galaxies have masses above $10^{10} M_\odot$, one would require a conversion efficiency of mass into UHE cosmic rays of only 0.6% (assuming that the source of the UHE cosmic rays produces these protons with left-handed and right-handed down valence quarks in equal numbers), which seems quite reasonable: even ordinary hydrogen fusion has a 0.7% efficiency for conversion of mass into energy, and black hole models can give mass-energy conversion efficiencies up to 42%.

The sources for a $3 \times 10^{20} \text{ eV}$ proton which are allowed by the Hillas criterion, namely that an allowed source must satisfy $BR \sim 10^{18} \text{ G cm}$, where B is the magnetic field and R is the size of the region with this roughly constant field, are radio galaxy lobes and AGNs, as is well-known. However, heretofore, these sources have been eliminated on the grounds that they are too far away. If the CMBR is actually a pure $SU(2)_L$ field, then such sources are perfectly acceptable.

9.c Cosmic Ray Physicists Have Once Again Seen New Fundamental Physics

Cosmic ray physicists have in the past made great discoveries in fundamental physics: in 1932, the discovery of the positron ([3], reprinted in [4]); in 1937 the discovery of muons; and in 1947, the discovery of pions and kaons ([4], pp. 50–51). Positrons were the first examples of anti-matter, and finding them deservedly got Carl Anderson the Noble Prize. Muons are elementary particles according to the Standard Model, as are s-quarks, which made their first appearance in kaons. The first s-quark baryons, the Λ , the Ξ^\pm , and the Σ^+ particles, were first detected by cosmic ray physicists, in 1951, in 1952, and in 1953, respectively ([4], pp. 50–51). But it has been almost half a century since cosmic ray physicists have made

recognized fundamental discoveries. I believe that the discovery of the UHE cosmic rays are an *unrecognized* discovery of fundamental importance: the observation of these particles demonstrates that the CBR is not an electromagnetic field, but rather the pure SU(2) component of an electromagnetic field.

The expression of many theorists concerning UHE cosmic rays, that these particles simply *must* be merely a local phenomena, reminds me of Herzberg's 1950 remark on the observation that CN molecules in interstellar clouds appeared to be in a heat bath of around 2.3 K: "which has of course only a very restricted meaning." ([11], p. 497), by which he mean that the 2.3 heat bath was merely a phenomena local to the molecular cloud.

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